Mathematics

Constructed Response Items
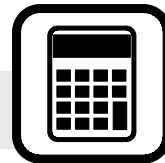
Measures of Student Learning: NC's Common Exams

Released Fall 2012

Table of Contents

The questions you read next will require you to answer in writing.

1. **Write your answers on separate paper.**

2. **Be sure to write your name on each page.**

1   The chart below shows the amount of insulin in a person's bloodstream after a certain amount of time, *t*.

t (minutes)	3	15	24	45
Units of Insulin	8.6	4.9	3.1	1.0

Create a best fit exponential function to answer the questions.

- To the nearest tenth, how many units of insulin are in the person's bloodstream at $t = 0$?

- To the nearest percent, what is the absolute value of the percent change per minute of insulin?

2   A geologist is analyzing the erosion of a coastline over the past five years. The table below shows the relationship.

| **Time** (years)	1	2	3	4	5
Cumulative Erosion (feet)	1.01	2.81	6.51	10.14	16.32

- Does a linear, exponential, or power function best fit the data? Explain.

- Write the equation of the function that best models the data.

- Using the equation created, how much erosion can be expected after 8 years?

3   A restaurant determined that the calories of its meals are normally distributed. The mean is 680 calories with a standard deviation of 63 calories.

- Give a range of calories, centered on the mean, that includes 95% of the restaurant's meals.

- The cook creates a meal with 617 calories. What percentage of meals has more calories than this meal?

4   The table below shows the cumulative frequency of student achievement scores for a particular class.
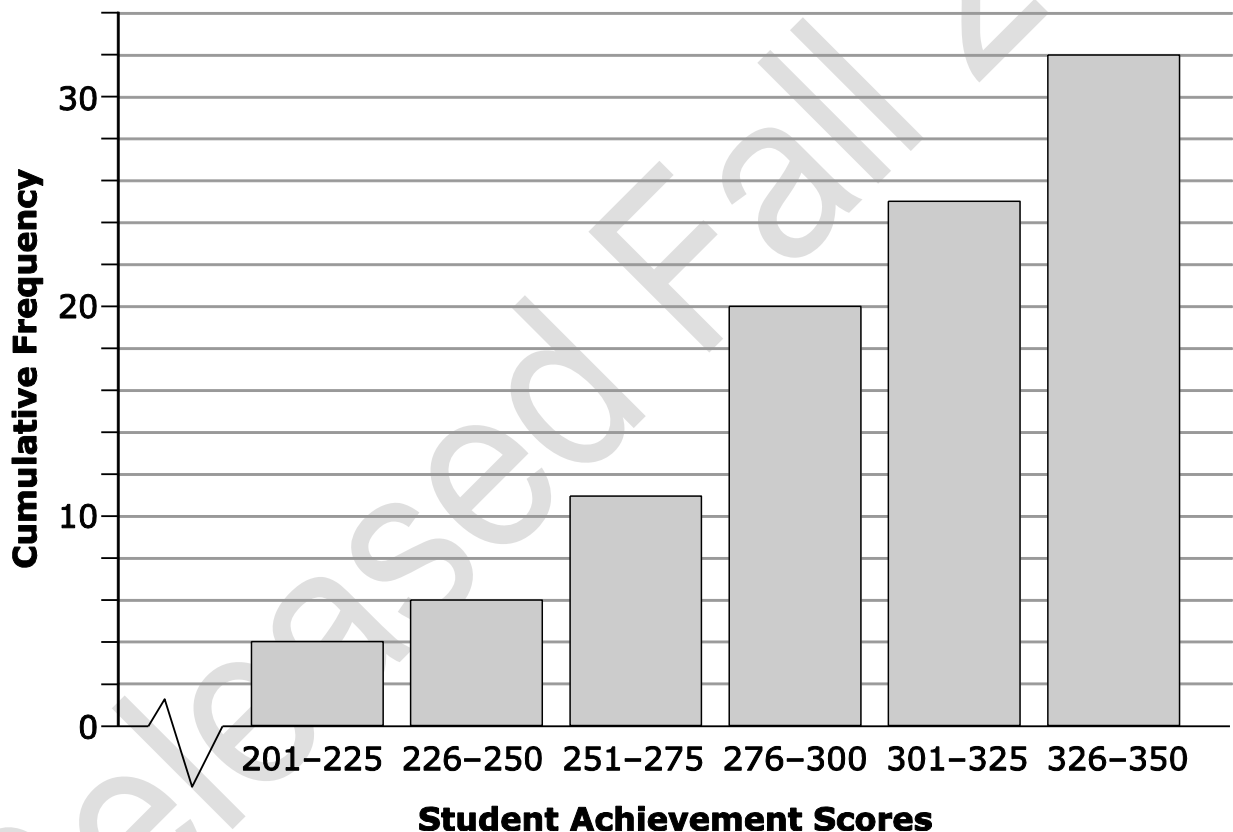
- Which 25-point interval contains the median achievement score?

- How many students scored at least a 276 on the achievement test?

5   A professor of a statistics class has the following scores for her students: 296, 332, 290, 308, 343, 371, 336, 384, 361, 350.

  •   If 10 points are added to each score, what is the new mean and standard deviation?

6   Use the piecewise function below to answer each question.

$$h(x) = \begin{cases} {}^{-}2x^2 + 5x + 10 & \text{for } {}^{-}4 \le x < 3 \quad \text{Step 1} \\ 3x + 2 & \text{for } \quad 3 \le x < 7 \quad \text{Step 2} \\ \sqrt{2x - 5} & \text{for } \quad 7 \le x < 16 \text{ Step 3} \end{cases}$$

  •   What is the range for step 1?
  •   What is the domain for the entire function?
  •   What is $h(10.5)$?

7   For the following problem, round each answer to the nearest hundredth.

  •   Write the equation of the power function that passes through the points (1, 6) and (3, 14).

  •   Based on the above function, what is the value of x when $y = 8$?

8   Power functions can be written in the form $f(x) = ax^b$, where $b > 1$.

  •   Write an equation for an odd, positive-integer power function of this form.

  •   Graph the equation including enough of the domain to show the graph's important features.

1. The chart below shows the amount of insulin in a person's bloodstream after a certain amount of time, t.

t (minutes)	3	15	24	45
Units of Insulin	8.6	4.9	3.1	1.0

Create a best fit exponential function to answer the questions.

- To the nearest tenth, how many units of insulin are in the person's bloodstream at $t = 0$?

- To the nearest percent, what is the absolute value of the percent change per minute of insulin?

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Calculates a number for the units of insulin at $t = 0$ that fits with the data; calculates an absolute value for the percent change that fits the data

2. A geologist is analyzing the erosion of a coastline over the past five years. The table below shows the relationship.

Time (years)	1	2	3	4	5
Cumulative Erosion (feet)	1.01	2.81	6.51	10.14	16.32

- Does a linear, exponential, or power function best fit the data? Explain.
- Write the equation of the function that best models the data.
- Using the equation created, how much erosion can be expected after 8 years?

Score 0   No response or the response does not address the task

Score 1   Fulfills only 1 of 3 requirements of a level 3 performance

Score 2   Fulfills 2 of 3 requirements of a level 3 performance

Score 3   Indicates the type of function that best models the data and explains its meaning; writes an equation that best models the data; calculates the amount of erosion expected after 8 years using the written equation

3. A restaurant determined that the calories of its meals are normally distributed. The mean is 680 calories with a standard deviation of 63 calories.

- Give a range of calories, centered on the mean, that includes 95% of the restaurant's meals.

- The cook creates a meal with 617 calories. What percentage of meals has more calories than this meal?

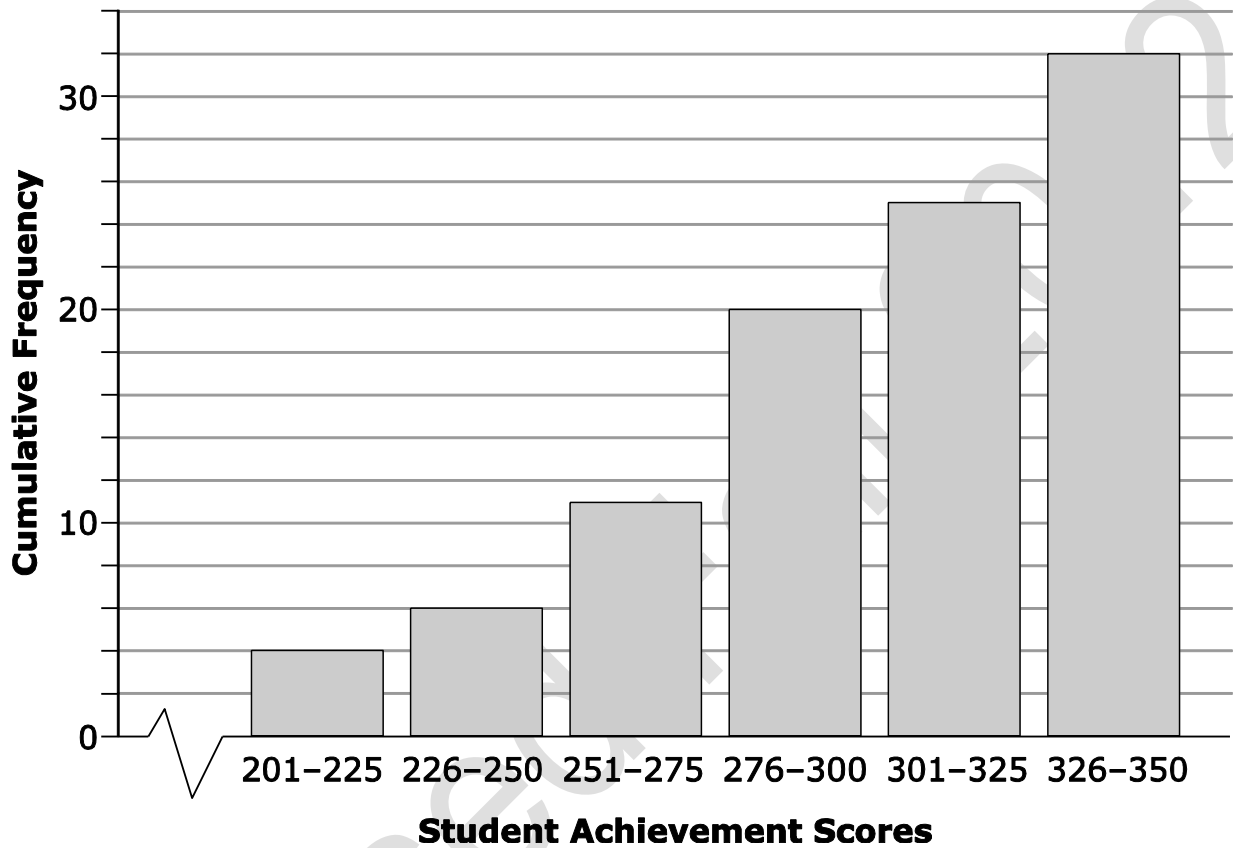**Score 0**   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Determines a range of calories centered around the mean that 95% of the meals will fall in; determines the percentage of meals that contain more than 617 calories

4. The table below shows the cumulative frequency of student achievement scores for a particular class.

Student Achievement Scores

- Which 25-point interval contains the median achievement score?
- How many students scored at least a 276 on the achievement test?

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Identifies an interval that contains the median of the data; determines how many students scored at least 276 points

5. A professor of a statistics class has the following scores for her students: 296, 332, 290, 308, 343, 371, 336, 384, 361, 350.

- If 10 points are added to each score, what is the new mean and standard deviation?

Score 0 No response or the response does not address the prompt

Score 1 Determines the mean OR standard deviation for the new data

Score 2 Determines the mean AND standard deviation for the new data

6. Use the piecewise function below to answer each question.

$$h(x) = \begin{cases} {}^-2x^2 + 5x + 10 & \text{for } {}^-4 \le x < 3 & \text{Step 1} \\ 3x + 2 & \text{for } 3 \le x < 7 & \text{Step 2} \\ \sqrt{2x - 5} & \text{for } 7 \le x < 16 & \text{Step 3} \end{cases}$$

- What is the range for step 1?
- What is the domain for the entire function?
- What is $h(10.5)$?

Score 0 No response or the response does not address the prompt

Score 1 Fulfills 1 of 3 requirements of a level 3 performance

Score 2 Fulfills 2 of 3 requirements of a level 3 performance

Score 3 Determines the range for step 1; determines the domain for the entire function; calculates the value of $h(10.5)$

7. For the following problem, round each answer to the nearest hundredth.

- Write the equation of the power function that passes through the points (1, 6) and (3, 14).

- Based on the above function, what is the value of *x* when *y* = 8?

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Provides an equation for the power function that passes through the points given; calculates a value for *x* when *y* = 8 using
the written equation

8. Power functions can be written in the form $f(x) = ax^b$, where $b > 1$.

- Write an equation for an odd, positive-integer power function of this form.

- Graph the equation including enough of the domain to show the graph's important features.

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Writes an equation for an odd, positive-integer power function; graphs an image of the chosen function

| QID	Discrete Answers for Student Responses (* Student answers may vary.)
1	Score Point 1: * Score Point 2: *
2	Score Point 1: power function with explanation from student Score Point 2: * Score Point 3: *
3	Score Point 1: 554 calories to 806 calories Score Point 2: 84%
4	Score Point 1: 276–300 interval Score Point 2: 21 students
5	Score Point 1: mean score of 347.1 Score Point 2: standard deviation of 31.5
6	Score Point 1: [-42, 13.125] Score Point 2: [-4, 16) Score Point 3: 4
7	Score Point 1: * Score Point 2: *
8	Score Point 1: * Score Point 2: *

The questions you read next will require you to answer in writing.

1. **Write your answers on separate paper.**

2. **Be sure to write your name on each page.**

1   A farmer has 600 yards of fence. He will use some of the fence to enclose a rectangular area. He will use the rest to divide the area into two congruent rectangles, as shown below.
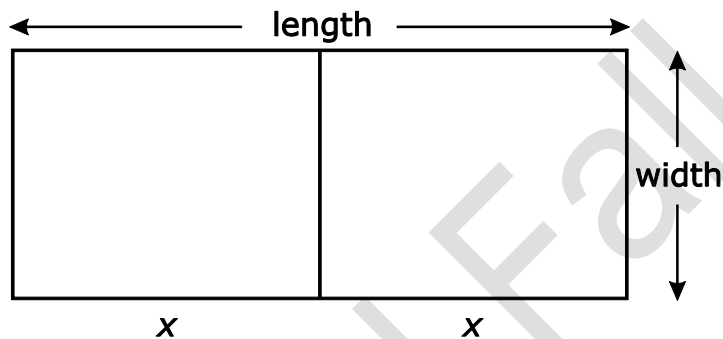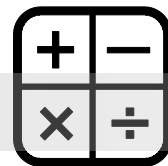

- What is the value of *x* that results in the largest area?
- What is the largest area that the farmer can enclose?
- What are the length and the width of the outer fence that will produce the largest total area?
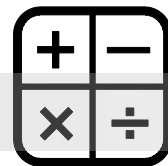
2   Suppose that Kyle has $1,500 to invest. His investment will earn an interest rate of 8.25% compounded continuously.

- To the nearest cent, what will be the value of Kyle's investment after 6 years?
- To the nearest tenth, how long will it take for Kyle's investment to grow to $3,000?
- To the nearest tenth, what interest rate would be needed to triple Kyle's investment in 15 years?

3   The function $f(t) = 36(0.50)^{\frac{1}{5,730}t}$ models the amount of carbon-14, in mg, remaining in a sample t years after the year 1200.

- What amount of carbon-14 was present in the sample in the year 1200?
- What is the meaning of the coefficient of t in the context of the problem?

4   An apple orchard puts 6 apples in each basket it sells. Each apple weighs within 5% of 8 ounces.

- Write a single inequality using absolute value that can be used to find the minimum total weight and the maximum total weight of the apples in each basket.
- What are the minimum total weight and the maximum total weight of the apples in each basket?

5   A company produces two types of bicycle tires: mountain bike tires and racing tires.

    I.   The expected demand for mountain bike tires is at least 100 tires per week, and the expected demand for racing tires is at least 80 tires per week.

    II.   The company can produce no more than 200 mountain bike tires per week, and no more than 180 racing tires per week.

    III.   The company sells a maximum of 225 tires per week, and makes a net profit of \$5 per mountain bike tire and \$9 per racing tire.

- Write a system that models the constraints on bicycle tire production. Define the variables you use.

- How many of each tire should the company produce in order to maximize profit?

1. A farmer has 600 yards of fence. He will use some of the fence to enclose a rectangular area. He will use the rest to divide the area into two congruent rectangles, as shown below.
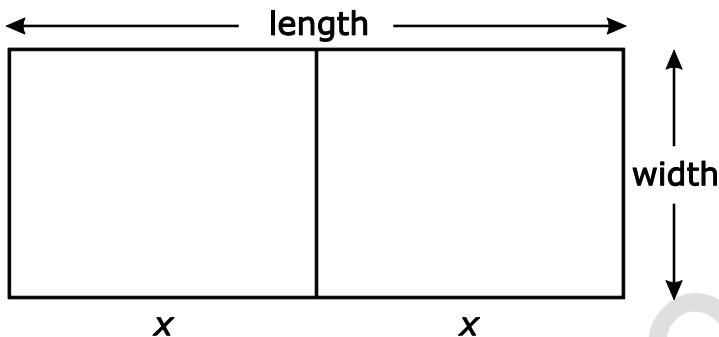
- What is the value of *x* that results in the largest area?
- What is the largest area that the farmer can enclose?
- What are the length and the width of the outer fence that will produce the largest total area?

Score 0	No response or the response does not address the prompt
Score 1	Fulfills only 1 of 3 requirements of a level 3 performance
Score 2	Fulfills 2 of 3 requirements of a level 3 performance
Score 3	Determines a value for *x* that results in the largest area; determines a value for the largest area enclosed; provides a value for the dimensions of the fence that will create the largest area

2. Suppose that Kyle has $1,500 to invest. His investment will earn an interest rate of 8.25% compounded continuously.

- To the nearest cent, what will be the value of Kyle's investment after 6 years?
- To the nearest tenth, how long will it take for Kyle's investment to grow to $3,000?
- To the nearest tenth, what interest rate would be needed to triple Kyle's investment in 15 years?

Score 0	No response or the response does not address the prompt
Score 1	Fulfills only 1 of 3 requirements of a level 3 performance
Score 2	Fulfills 2 of 3 requirements of a level 3 performance
Score 3	Determines a value for the investment after 6 years; determines how long it will take the investment to grow to $3,000; determines the interest rate needed for the investment to triple in 15 years

3. The function $f(t) = 36(0.50)^{\frac{1}{5,730}t}$ models the amount of carbon-14, in mg, remaining in a sample t years after the year 1200.

    - What amount of carbon-14 was present in the sample in the year 1200?

    - What is the meaning of the coefficient of t in the context of the problem?

Score 0 No response or the response does not address the prompt

Score 1 Fulfills only 1 of 2 requirements of a level 2 performance

Score 2 Identifies a value for the amount of carbon-14 present in year 1200; explains the meaning of the coefficient of t in the context of the problem

4. An apple orchard puts 6 apples in each basket it sells. Each apple weighs within 5% of 8 ounces.

    - Write a single inequality using absolute value that can be used to find the minimum total weight and the maximum total weight of the apples in each basket.

    - What are the minimum total weight and the maximum total weight of the apples in each basket?

Score 0 No response or the response does not address the prompt

Score 1 Fulfills only 1 of 2 requirements of a level 2 performance

Score 2 Provides an inequality that accurately models the scenario; determines the minimum and maximum weights for the baskets based on the written equation

5. A company produces two types of bicycle tires: mountain bike tires and racing tires.

    IV.   The expected demand for mountain bike tires is at least 100 tires per week, and the expected demand for racing tires is at least 80 tires per week.

    V.   The company can produce no more than 200 mountain bike tires per week, and no more than 180 racing tires per week.

    VI.   The company sells a maximum of 225 tires per week, and makes a net profit of $5 per mountain bike tire and $9 per racing tire.

- Write a system that models the constraints on bicycle tire production. Define the variables you use.
- How many of each tire should the company produce in order to maximize profit?

Score 0   No response or the response does not address the prompt

**Score 1**   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Writes a system of equations that models the constraints on tire production; determines a value for the number of each type of tire the company should produce to maximize profit
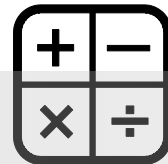
| QID	Discrete Answers for Student Responses (* Student answers may vary.)
1	Score Point 1: 75 yards Score Point 2: 15,000 yards^2 Score Point 3: length = 150 yards, width = 100 yards
2	Score Point 1: $2,460.75 Score Point 2: 8.4 years Score Point 3: 7.3 %
3	Score Point 1: 36 mg Score Point 2: *
4	Score Point 1: * Score Point 2: *
5	Score Point 1: * Score Point 2: *
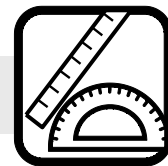
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      1. Write your answers on separate paper.
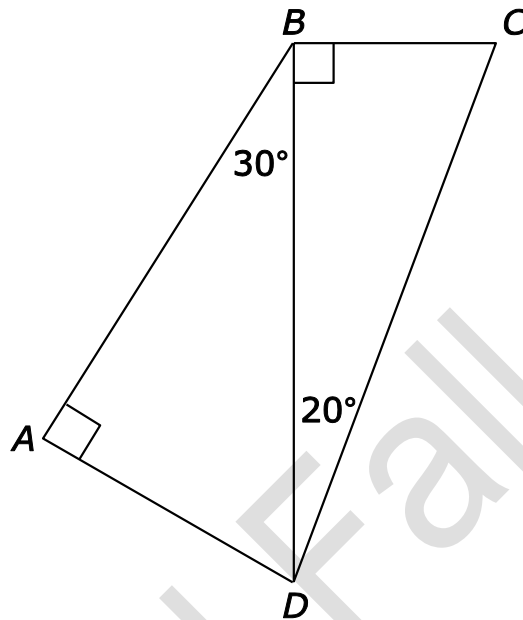
      2. Be sure to write your name on each page.

1   Circle *M* is shown below.
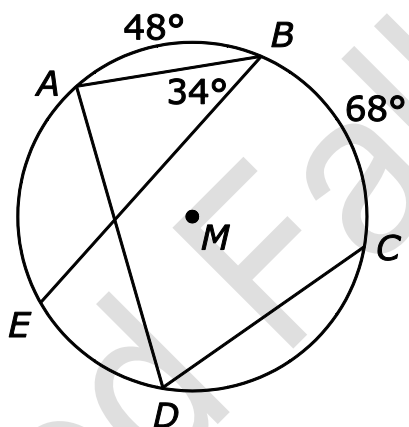
-     What is the measure of ∠*ADC* ?
-     What is the measure of *AE* ?

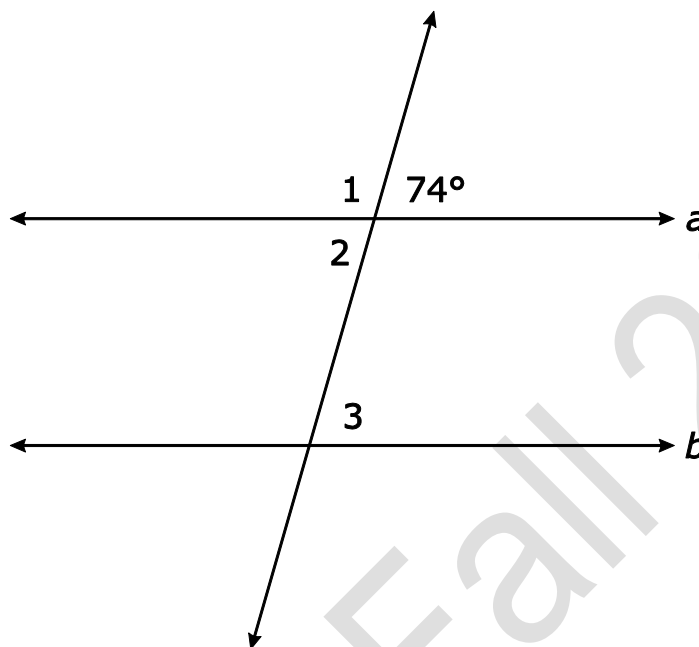2   In quadrilateral *ABCD*, *BD* = 14 cm.

To the nearest tenth,

- What is the measure of $\overline{AD}$?
- What is the measure of $\overline{BC}$?
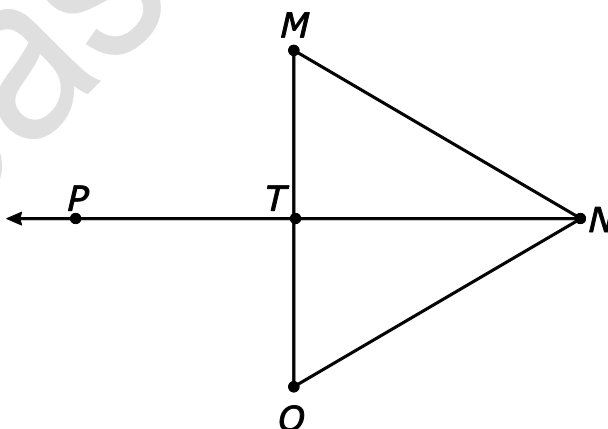- What is the measure of $\overline{CD}$?

3   In the diagram below, $a \parallel b$.

What are the measures of $\angle 1$, $\angle 2$, and $\angle 3$? Explain how each angle measure is determined.
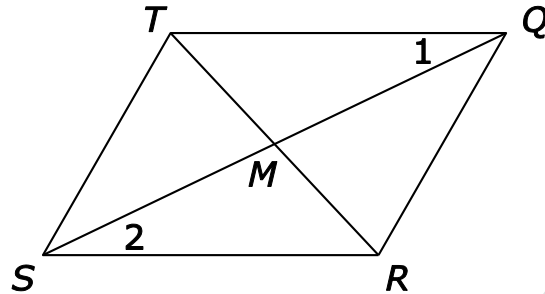
4   In the diagram of $\square MNO$, $\overrightarrow{NP}$ is drawn, $\angle MTP \cong \angle OTP$ and $\overline{MT} \cong \overline{OT}$.

Using the diagram, write a proof showing that $\overline{NT}$ bisects $\angle MNO$.

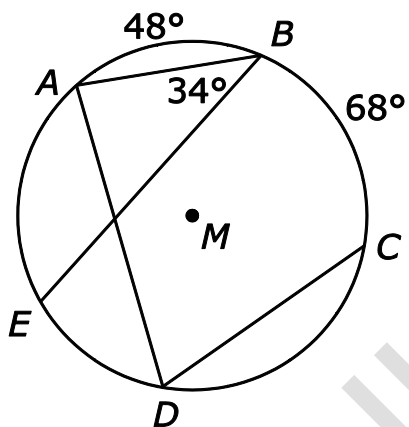5   In the figure below, $\overline{MS} \cong \overline{MQ}$ and $\angle 1 \cong \angle 2$.

Based on the given facts, write a proof showing *QRST* is a parallelogram.

1. Circle *M* is shown below.

- What is the measure of ∠*ADC* ?

- What is the measure of *AE* ?

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Determines a value for the measure of angle *ADC*; determines a value for the measure of arc *AE*

2. In quadrilateral *ABCD*, *BD* = 14 cm.

To the nearest tenth,

- What is the measure of $\overline{AD}$?
- What is the measure of $\overline{BC}$?
- What is the measure of $\overline{CD}$?

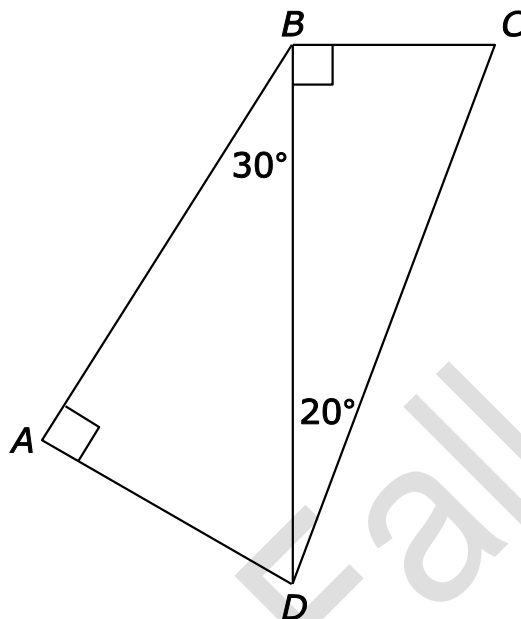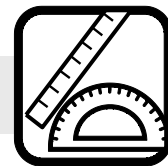**Score 0** No response or the response does not address the prompt
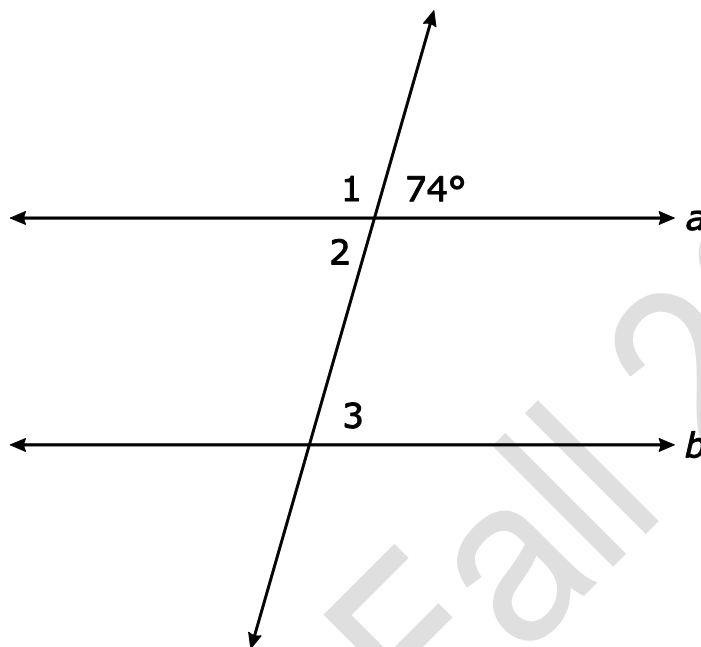
Score 1 Fulfills only 1 of 3 requirements of a level 3 performance

Score 2 Fulfills 2 of 3 requirements of a level 3 performance

Score 3 Determines the length of line segment *AD*; determines the length of line segment *BC*; determines the length of line segment *CD*

3. In the diagram below, $a \parallel b$.

What are the measures of $\angle 1$, $\angle 2$, and $\angle 3$? Explain how each angle measure is determined.

Score 0	No response or the response does not address the prompt
Score 1	Fulfills only 1 of 3 requirements of a level 3 performance
Score 2	Fulfills 2 of 3 requirements of a level 3 performance
Score 3	Determines the measure of angle 1 and explains how it is determined; determines the measure of angle 2 and explains how it is determined; determines the measure of angle 3 and explains how it is determined

4. In the diagram of $\square MNO$, $\overrightarrow{NP}$ is drawn, $\angle MTP \cong \angle OTP$ and $\overline{MT} \cong \overline{OT}$.
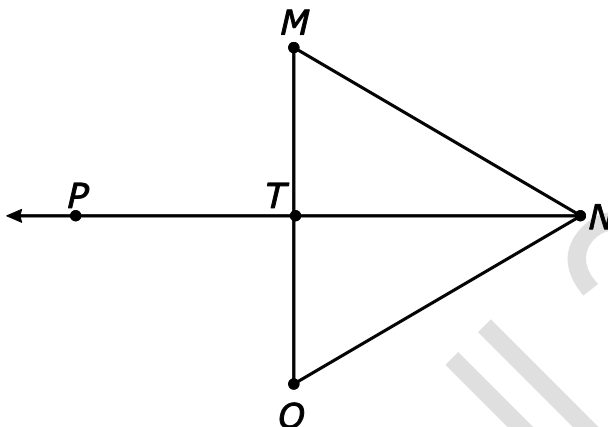
Using the diagram, write a proof showing that $\overline{NT}$ bisects $\angle MNO$.

Score 0	No response or the response does not address the prompt
Score 1	Fulfills only 1 of 3 requirements of a level 3 performance
Score 2	Fulfills 2 of 3 requirements of a level 3 performance
Score 3	Provides a complete proof which includes logical steps with accurate notation and wording of theorems, postulates, etc.; includes relevant, correct statements and reasons; sequences statements logically and aligns reasons accurately to their statements

5. In the figure below, $\overline{MS} \cong \overline{MQ}$ and $\angle 1 \cong \angle 2$.

Based on the given facts, write a proof showing *QRST* is a parallelogram.

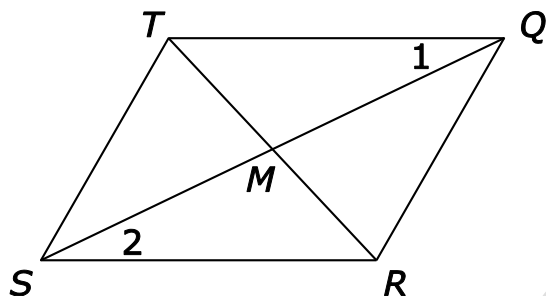|	
Score 0	No response or the response does not address the prompt
Score 1	Fulfills only 1 of 3 requirements of a level 3 performance
Score 2	Fulfills 2 of 3 requirements of a level 3 performance
Score 3	Provides a complete proof, which includes logical steps with accurate notation and wording of theorems, postulates, etc.; includes relevant, correct statements and reasons; sequences statements logically and aligns reasons accurately to their statements
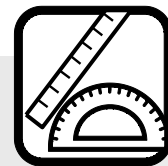
QID	Discrete Answers for Student Responses (* Student answers may vary.)
1	Score Point 1: 58° Score Point 2: 68°
2	Score Point 1: 7.0 cm Score Point 2: 5.1 ° or any equivalent/rounded value Score Point 3: 14.9 ° or any equivalent/rounded value
3	Score Point 1: 106 ° Score Point 2: 74° Score Point 3: 74°
4	Score Point 1: * Score Point 2: * Score Point 3: *
5	Score Point 1: * Score Point 2: * Score Point 3: *

The questions you read next will require you to answer in writing.

- **Write your answers on separate paper.**

- **Be sure to write your name on each page.**

1   The general equation of an ellipse is shown below.

$$9x^2 - 54x + 25y^2 - 100y - 44 = 0$$

- Write an equivalent standard equation for the ellipse.
- Describe what the coefficients of x^2 and y^2 in the standard equation tell about this ellipse.

2   A child is pulling a sled through the snow with a force of 20 Newtons at an angle of 40°.

- To the nearest tenth, what is the vertical component of the force?
- To the nearest tenth, what is the horizontal component of the force?

3   The function $P(t) = 300e^{(0.038t)}$ models the number of bacteria in a population after t minutes.

- What is the meaning of the coefficient of e in the context of the problem?
- What is the meaning of the coefficient of t in the context of the problem?

4   A person is on a ride at a carnival. The table below shows approximately how high the person is off the ground after t seconds.

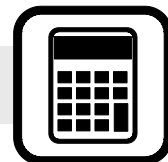| Seconds (t)	Feet off the Ground (h)
0	3
10	14
20	25
30	33
40	25
50	14
60	3

- Write an equation of the sine function that **best** fits the data.
- What is the meaning of the constant term in the equation you derived?

5   Scientists estimated the number of mosquitoes living in an area in different years.

Year	Mosquitoes (in thousands)
1960	6
1970	8
1980	12
1990	18
2000	34
2010	42

The scientists then decided to use an exponential best-fit model to predict the number of mosquitoes that will be in the area in 2020.

- Write an equation that the scientists used to make their prediction.
- Use your equation to predict how many mosquitoes will be living in the area in 2020?

6   A window maker uses the graph of a rose curve to create a pattern in stained glass.

- If the window maker uses the equation $r = 8\cos 4\theta$ to represent the curve, what is the maximum petal length?

- How many petals does the graph have?

- Explain your answers.

7   After driving 30 minutes at a constant rate, a mailman is 200 miles from his home. After 75 minutes, he is 150 miles from his home.

- Write a set of parametric equations that represent the mailman's distance from home over time.

8   Let $f(x) = 4^x$.

- Graph $f(x - 2) + 3$.
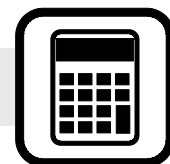
- Write a description of the transformation that occurred.

1. The general equation of an ellipse is shown below.

$$9x^2 - 54x + 25y^2 - 100y - 44 = 0$$

   - Write an equivalent standard equation for the ellipse.
   - Describe what the coefficients of x^2 and y^2 in the standard equation tell about this ellipse.

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Writes an equation for the standard version of the ellipse given; provides a description of the meaning of the coefficient of x^2 and y^2 in the standard equation written

2. A child is pulling a sled through the snow with a force of 20 Newtons at an angle of 40°.

   - To the nearest tenth, what is the vertical component of the force?
   - To the nearest tenth, what is the horizontal component of the force?

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Calculates the vertical component of the force given; calculates the horizontal component of the force given
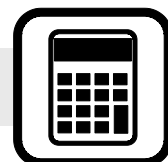
3. The function $P(t) = 300e^{(0.038t)}$ models the number of bacteria in a population after t minutes.

   - What is the meaning of the coefficient of e in the context of the problem?
   - What is the meaning of the coefficient of t in the context of the problem?

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Explains in context the meaning of the coefficient of e; explains in context the meaning of the coefficient of t

4. A person is on a ride at a carnival. The table below shows approximately how high the person is off the ground after *t* seconds.

Seconds (*t*)	Feet off the Ground (*h*)
0	3
10	14
20	25
30	33
40	25
50	14
60	3

- Write an equation of the sine function that **best** fits the data.

- What is the meaning of the constant term in the equation you derived?

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Writes a sine function that best fits the data; describes the meaning of the constant in the written equation

5. Scientists estimated the number of mosquitoes living in an area in different years.

Year	Mosquitoes (in thousands)
1960	6
1970	8
1980	12
1990	18
2000	34
2010	42

The scientists then decided to use an exponential best-fit model to predict the number of mosquitoes that will be in the area in 2020.

- Write an equation that the scientists used to make their prediction.
- Use your equation to predict how many mosquitoes will be living in the area in 2020?

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Writes an exponential best-fit equation for the data; predicts the number of mosquitoes living in the area in 2020 using the model

6. A window maker uses the graph of a rose curve to create a pattern in stained glass.

- If the window maker uses the equation $r = 8\cos 4\theta$ to represent the curve, what is the maximum petal length?
- How many petals does the graph have?
- Explain your answers.

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Calculates and explains what the petal length for the graph is; calculates and explains how many petals are created by the graph of the rose curve

7. After driving 30 minutes at a constant rate, a mailman is 200 miles from his home. After 75 minutes, he is 150 miles from his home.
   - Write a set of parametric equations that represent the mailman's distance from home over time.

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Creates a 2-equation parametrization that represents the problem; uses correct coefficients and constants to model the mailman's route home
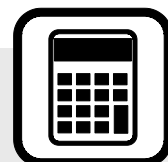
8. Let $f(x) = 4^x$.

   - Graph $f(x - 2) + 3$.

   - Write a description of the transformation that occurred.

Score 0   No response or the response does not address the prompt

Score 1   Fulfills only 1 of 2 requirements of a level 2 performance

Score 2   Graphs a correct transformation of the function; provides a correct description of the transformation that occurred

QID	Discrete Answers for Student Responses (* Student answers may vary.)
1	Score Point 1: $\frac{(x-3)^2}{25} + \frac{(y-2)^2}{9} = 1$ Score Point 2: *
2	Score Point 1: 12.9° Score Point 2: 15.3°
3	Score Point 1: * Score Point 2: *
4	Score Point 1: * Score Point 2: *
5	Score Point 1: * Score Point 2: *
6	Score Point 1: 8 with student explanation Score Point 2: 8 with student explanation
7	Score Point 1: * Score Point 2: *
8	Score Point 1: * Score Point 2: shifted to the right 2 units and up 3 units