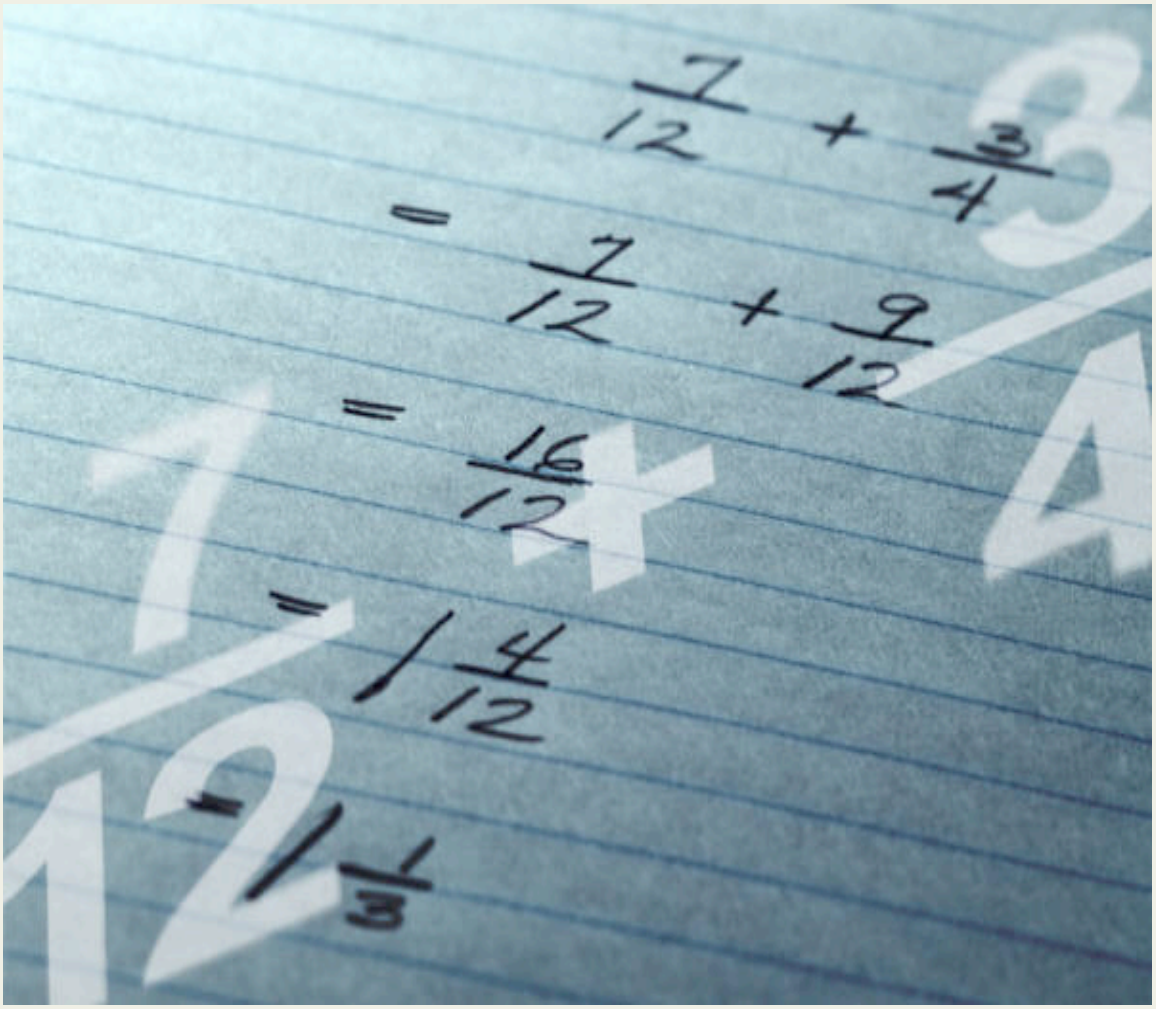
Fifth Grade

Multiplying and Dividing Fractions



North Carolina Department of Public Instruction

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Overview

The implementation of the Common Core State Standards in Mathematics (CCSSM) is both an exciting and anxious time for teachers around the country. Part of the excitement is the CCSS inclusion of both the Content Standards and the Standards for Mathematical Practice. The Standards for Mathematical Practice provide a foundation for the process skills that all K-12 students should be developing during every lesson.

Overview of the Unit

The purpose of this document is to provide teachers with a set of lessons that are standards-based and align with the CCSS Content Standards and Standards for Mathematical Practice. By standards-based, we mean that students are learning mathematics by exploring mathematically-rich tasks and sharing strategies, ideas, and approaches with one another. During these lessons, the teacher's role is to truly facilitate learning by posing a task, asking questions that guide students' understanding, and assess students' mathematical understanding.

The phases of each lesson are:

- **Engage-** Students open the lesson by engaging in a brief activity to build upon students' prior knowledge.
- **Explore-** Students explore a mathematically rich task or activity that includes the main mathematical goals. During this phase, the teacher may model how to play a game or do an activity, but should not model or over teach strategies or procedures.
- **Explain-** Students discuss strategies and mathematical ideas from the Explore phase. The teacher may teach content and emphasize concepts or strategies here.
- **Elaborate-** Students complete a follow-up activity or task that extends their work from Explore and the discussion of concepts in Explain.
- **Evaluation of Students**
  - **Formative Assessment-** How can the teacher assess students during the lesson?
  - **Summative Assessment-** How can the teacher assess students' work after the lesson?

Resources on the Common Core

This document is only a starting resource as teachers begin implementing the CCSS and the Standards for Mathematical Practice. The North Carolina Department of Public Instruction has also written Unpacking Documents available at http://www.ncpublicschools.org/acre/standards/ support-tools/ . These unpacking documents provide specific descriptions of each standard as well as examples.

This project was directed by Dr. Drew Polly at UNC Charlotte. Educators who collaborated to create these documents are Gail Cotton, Ryan Dougherty, Tricia Esseck, Marta Garcia, Tery Gunter, and Kayonna Pitchford along with the DPI staff.

Unit Overview: Grade 5

Mathematical Goals

In this unit, students will:

- Apply and extend their understanding of whole number operations (specifically multiplication and division) to include fractional numbers.
- Develop visual models for multiplying and dividing fractions (area models, arrays, bar and number line models)
- Make use of benchmarks and other strategies to determine reasonableness of the solutions when operating with fractions
- Explore the relationship between two numbers and their product
- Interpret and create story contexts related to appropriate operations
- Use knowledge of fractions, equivalences and the inverse relationship between multiplication and division to solve problems and develop strategies for operating on fractions.

Lessons in Unit

This lesson is intended to be taught in the second half of the year at the end of the unit on fractions.

Lesson(s)	Title and Description
1	**Servings at the Fifth Grade Ice Cream Party**: Students will solve story problems about ice cream cake servings at a fifth grade party and goal targets for a contest. **Materials:** student sheets, colored pencils, graph paper, fraction bars
2 part: 1 and part: 2	**Collecting Recyclables**: Students will solve story problems about collecting recyclables. **Materials:** student sheets, colored pencils, graph paper, fraction bars
3	**Servings at the Fifth Grade Ice Cream Party**: Students will solve story problems about ice cream cake servings at a fifth grade party and goal targets for a contest. **Materials:** student sheets, colored pencils, graph paper, fraction bars
4	**Tiling the Walls of the Art Room**: Students will work within a story context about fifth grade students tiling the walls of the new art room at their school. They will make use of area models specifically arrays to make sense of the multiplication of unit fractions. **Materials:** student sheets, graph paper, tiles, white paper
5	**Who Won? The Lance School Relay Marathon**: During this lesson the students will make use of a linear model (number lines) and bar models. **Materials:** adding machine tape, student sheets, rulers
6	**Sharing Prizes at the School Carnival:** Students work with various models of fractions as they multiply and divide fractions. **Materials:** student sheets, fraction bars

Assessment in this Unit
Students can be assessed in the following ways:

- Includes observation, anecdotal notes, and student work samples
- Each lesson will include opportunities for formative assessment.
- Each lesson will include questions for teachers to consider during their observations.
- Student work samples will be included as benchmark samples.

Common Core Standards addressed in this unit:
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.5 Interpret multiplication as scaling (resizing), by:
a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1
a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.
b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) =

20 because 20 × (1/5) = 4.

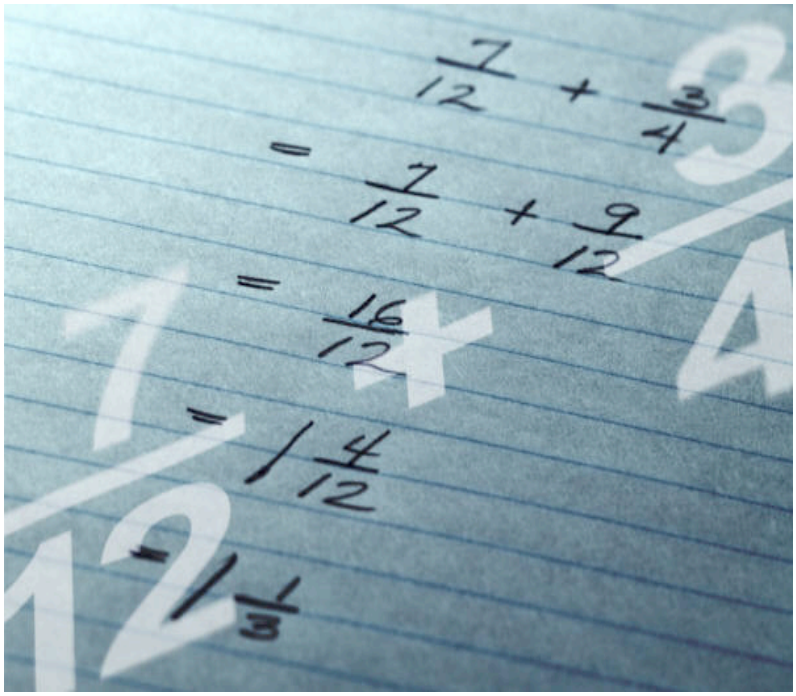
c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

[1]Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

Emphasized Standards for Mathematical Practice

In this unit all of the Standards for Mathematical Practice are addressed.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning



Lesson 1: Servings at the Fifth Grade Ice Cream Party

Overview and Background Information

Students will solve story problems about ice cream cake servings at a fifth grade party. They will incorporate the use of fraction number lines. Students will begin to develop the following generalization: when you multiply two whole numbers, neither of which is zero, your product is always equal to or greater than each of the factors; when you multiply a fraction less than 1 by another fraction less than 1, the product is always less than either factor.

Mathematical Goals	By the end of the lesson students will: • Decompose fractions additively and relate repeated addition to multiplication (4 x 1/3 = 1/3+1/3+1/3+1/3) • Relate partitioning and sharing contexts to fractions (division of numerator by the denominator) • Understand that the resulting product of a whole number and a fraction less than one will be greater than the fraction and less than the whole number factor • Interpret and create visual models for multiplying fractions (number lines and fraction bars) • Interpret and create story contexts for multiplying fractions
Common Core State Standards	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.3** Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.) **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
Emphasized Standards for Mathematical Practice	**2. Reason abstractly and quantitatively** In this cluster of lessons students will have ample opportunities to engage in: "Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects." **7. Look for and make use of structure.** In this cluster of lessons students will have ample opportunities to engage in: "Mathematically proficient students look closely to discern a pattern or structure."

Prior Knowledge Needed	• Understand that fractions are numbers that represent quantities less than a whole • Understand the meaning of multiplication with whole numbers • Familiarity with number lines and bar models • Able to decompose fractions additively (3/4= 1/4+1/4+1/4) • **4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number
Vocabulary	factor, product, fraction, fraction less than one, number Line
Materials	colored pencils, graph paper, fraction bars
Resources	*This lesson will be extended in the Lance School Marathon Lessons later in this unit.

Tasks in the Lesson

Engage 8-10 minutes

During this part of the lesson, students will be introduced to the story context for the next two lessons. They will have an opportunity to make estimates for the types of problems they will encounter in the lessons.

- *About How Much?* Begin by engaging students in an estimation routine. This launch will be used as an assessment and as an opportunity to model how estimating will be useful as students solve problems.
- Let students know that they will be solving problems about goals that students made for a recycling drive and servings at an ice cream party
- Share sheet "Ice Cream". Reveal and discuss one problem at a time. Ask students to predict how much ice cream was served and to explain their thinking.
- Ask students how they are making their estimates. Take note of strategies and misconceptions.

  Questions to pose:

  What models or pictures were you visualizing?
  How did you decide if your estimate would be more or less than 1?
  Which estimates were not reasonable and why?

Note: During this phase of the lesson it is not necessary to identify that the story contexts were multiplication problems or to record any equations. This is an opportunity for students to recognize the importance of the strategies they use to estimate and how the strategies allow them to check the reasonableness of their answers.

Explore 25-30 minutes

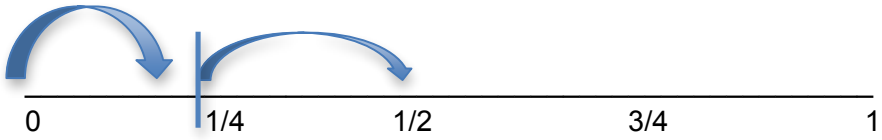
During this phase of the lesson, students will be introduced to a number line model for solving problems that involve thinking about groups of unit fractions and then groups of non-unit fractions. They will be developing the idea that (a) groups of size b/d is an accumulation of b/d +b/d ….

Introduce the story context: A fifth grade class was collecting recyclables for a drive and then celebrated their success with an ice cream party. We are going to be solving problems about how much ice cream

was served at the party. We will be using number lines representations to solve the problems. Let's look a this first story context together: (Write the context on a chart or board)

Amy helped serve mint chocolate chip ice cream. Her booth had paper bowls that hold $\frac{1}{4}$ cup servings. In the first five minutes, Amy served 3 students their bowls of ice cream. How much ice cream has she served?

Introduce a fraction number line. Draw a number line that has two tick marks for 0 and 1. Ask students how they could represent ¼ on the number line. How many ¼ would there be on this number line? Students may think of the idea that ¼ is ½ of ½, so they would mark ½. Then they would split the space between 0 and ½ in half again and mark that space as ¼.



Engage students in discussing how they would partition the number line into equal sized pieces. Label tick marks on the number line showing 0/4 ¼ 2/4 ¾ and 4/4. Point out the importance of the distances on the number line being equivalent.

Return to the story context. Ask students how they could represent the three scoops of size 1/5.

Mark 3 hops or moves of size 1/5 on the number line.

Ask students how they might be able to record the three hops represented on the number line.

Possible responses are :

¼ + ¼+ ¼ = ¾ or 3 groups of ¼ or 3 x ¼

Students work with partners or in small groups to solve the four ice cream problems on handout "The Fifth Grade Ice Cream Party".

As students work, ask them:

How does your number line model show the amount of ice cream that was served? How did you decide how to divide your number line? Which landmark numbers were useful as you found parts of the bar?

Pose questions that ask:

What is 4 groups of 1/5?

What is 2 groups of 1/3?

How does your drawing help someone see amount of ice cream that was served? Is there something you could add or do to your model to make it clearer?

Note: Problem 4 shifts the focus to multiplying a whole number by a non-unit fraction. Observe how students use their strategies for the first three problems in solving this problem. As students are working, begin selecting strategies and ideas for students to share in the whole group discussion.

Explain 17-20 minutes

During this phase of the lesson, the teacher will select and sequence student strategies and engage the class in a discussion that compares the methods that students used to solve the problems. The discussion will begin to assist students in relating the number of groups (as hops on the number line) and the size of the groups to the final product.

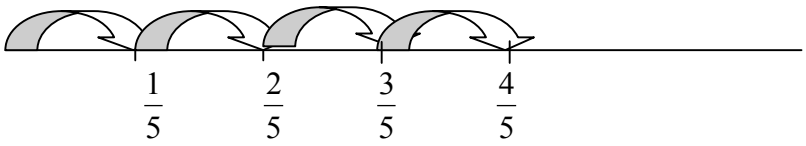
Bring students back together and discuss the ways in which students used the number lines to solve the problems.

Begin with problem 1.
One of the tables at the party has mint chocolate chip ice cream. The servings are 1/5 of cup. After five minutes, Ms. Coates had scooped out 4 servings. How much ice cream has she served?

As students share their ideas, listen for opportunities to talk about how the story contexts are asking for students to consider several groups of the same sized fraction.
Suggested questions to pose:

Is the amount of ice cream more or less than 1 full cup?
How do you know? Why is it less than ½ a cup? How does the number line model show you that there have been four servings scooped out?

Involve students in thinking about a form of notation: the whole number x unit fraction. (4 x 1/5)
Ask students what addition problem is equivalent to 4 x 1/5. (1/5 +1/5+1/5 +1/5 = 4/5).



Ask students what they notice about the size of the product related to the size of the two factors. Begin by asking what they notice about the product of 4x5=20(product is larger than both factors)

Then move to: 4 x 1/5 = 4/5 (the product is smaller than one factor (4) and larger than the other factor (1/5). Ask them to consider the other two problems. Chart these and begin to articulate a general statement about what they are noticing and why.

When you multiply a whole number (not 0) by a fraction less than 1, your product is smaller than the whole number and larger than the fraction.

Elaborate 15-20 minutes

Note: This can be part two of the lesson continued on the following day.

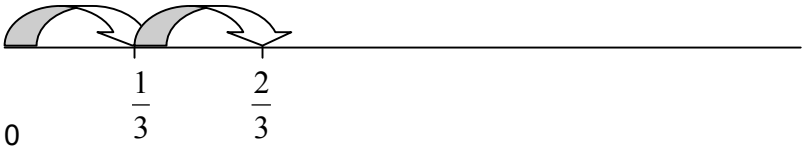
The last problem in the set asks students to consider a non unit fraction. Begin by asking the students how this problem is the same or different than the first three. (It has a whole number multiplied by a non-unit fraction)

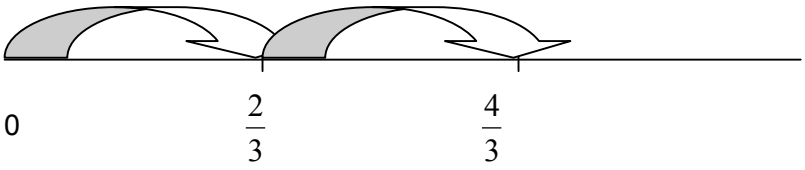
Ask students how Problem 4 is related to Problem 3.

At the sundae table, Lashay was serving mini marshmallows. She used 1/3 cup for each sundae. How much of the marshmallows has she used after making 2 sundaes?
During clean up time, Mr. Diaz found 2 gallon containers that were 2/3 full. How much ice cream was left?

*Problem 3 and 4 both have 2 groups of fractions that are thirds
*Problem 3 has a unit fraction 1/3. Problem 4 has 2/3.
*Problem 4 has a larger product than Problem 3

Ask students to assist you in drawing a number line that models problem 3 and then a number line that models problem 4.





Pose these questions:

Which number line has larger hops? How much larger are the hops?
How many 1/3 hops are there in 2/3? If you hop 4 hops of 1/3 where would you land on the number line? If you hop 2 hops of 2/3 where do you land?

As the students share responses to the questions, use the models of the number lines to make explicit the idea that although both problems involve thirds and 2 groups that problem 4 has hops that are double the size so the product is double the size. (Note: this work draws upon multiplication ideas of doubling with whole numbers and use of the distributive property.)

The following idea will continue to be explored in subsequent lessons but students should begin to consider how :

2 x 2/3 = 2 x (1/3 +1/3) = 2 x (2 x 1/3)

Discuss that this means 2 groups of 1/3 and 1/3 or four groups of 1/3.

Ask students to point out where there are two groups of 1/3 and 1/3 on the number line model for problem 4.

Pose questions such as: *How are 2 groups of 1/3 and 1/3 (2/3) the same as 4 groups of 1/3?*

Consider the following: How can we use the number line model to justify that 2 x 2/3 = 4 x 1/3?

Close by having students share any more ideas. Record these so that they are visible and so they can be revisited in the next lesson.

Students will be asked to rewrite problems 1 and 2 with non unit fractions and to compare the products of each of the problems. Students will be challenged to work on problems where the result is known and they must decide the number of servings that were served

At the sundae table, Lashay was serving mini marshmallows. She used 1/3 cup for each sundae. One bag only had 2/3 of a cup. How many sundaes can she top?

Mr. Diaz has 4/6 of a container of chocolate ice cream. He wants to serve 4 mini servings. What size should each serving be?

Evaluation of Students

Formative: Collect formative data and anecdotal notes through observations and questioning as students are working.

Summative: Students' work from the Elaborate section can be used throughout.

Plans for Individual Differences

Intervention:
If students are struggling they can use fraction manipulatives, such as fraction strips or fraction tiles. Sometimes those manipulatives, which are region models, are easier than number line models.

Extension:
For students in need of enrichment use an odd number of servings, such as 3 or 5 so that the fractional pieces are more complex.

Ice Cream

How much ice cream was served? Choose the closest estimate.

We served 4 boxes that had 12 ice cream cones each.

4 6 40 400

We served ½ a box that had 12 ice cream cones.

2 6 12 24

We had ½ a container of ice cream and ½ of what was in the container was scooped out. How much was scooped out?

1 container ½ of the container ¼ of the container

Name_____ Date_____

The Fifth Grade Ice Cream Party

Use fraction number lines to find out how much ice cream was served at the fifth grade party.

1. One of the tables at the party has mint chocolate chip ice cream. The servings are 1/5 of cup. After five minutes, Ms. Coates had scooped out 4 servings. How much ice cream has she served?

2. Hot fudge was a popular topping! At the end of the party, there were 3 containers left with ½ cup each of hot fudge. How much hot fudge was left?

3. At the sundae table, Lauren was serving mini marshmallows. She used 1/3 cup for each sundae. How much of the marshmallows has she used after making 2 sundaes?

4. During clean up time, Mr. Diaz found 2 gallon containers that were 2/3 full. How much ice cream was left?

Lesson 2: Part 1: Collecting Recyclables

Overview and Background Information

Part 1: *Students will solve story problems about goal targets for a contest. They will incorporate the use of fraction bars. Students will begin to develop the following generalization: when you multiply two whole numbers, neither of which is zero, your product is always equal to or greater than each of the factors; when you multiply a fraction less than 1 by another fraction less than 1, the product is always less than either factor. The problems will be limited to a whole number multiplied by a unit fraction in the first part of the lesson, to a whole number multiplied by a non-unit fraction in the second lesson and to a fraction multiplied by a fraction in the third part.*

Mathematical Goals	By the end of the lesson students will: • Decompose fractions additively and related repeated addition to multiplication (4 x 1/3 = 1/3+1/3+1/3+1/3) • Recognize multiplication of a fraction and a whole number as finding a part of group. • Relate partitioning and sharing contexts to fractions (division of numerator by the denominator) • Understand that the resulting product of a whole number and a fraction less than one will be greater than the fraction and less than the whole number factor. • Interpret and create visual models for multiplying fractions (number lines and fraction bars) • Interpret and create story contexts for multiplying fractions Essential Question: *What patterns do you notice in the products? How are the products related to the factors in the problems?*
Common Core State Standards	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.3** Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.) **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
Emphasized Standards for Mathematical Practice	**2. Reason abstractly and quantitatively.** In this cluster of lessons students will have ample opportunities to engage in: "Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects." **7. Look for and make use of structure.** In this cluster of lessons students will have ample opportunities to engage in: "Mathematically proficient students look closely to discern a pattern or structure."

Prior Knowledge Needed	• Understand that fractions are numbers that represent quantities less than a whole • Understand the meaning of multiplication with whole numbers • Familiarity with number lines and bar models • Able to decompose fractions additively (3/4= 1/4+1/4+1/4) • **4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number
Vocabulary	factor, product, whole number, fraction, fraction less than one, fraction bar
Materials	students sheets, colored pencils, graph paper, fraction bars
Resources	This lesson will be extended in the Lance School Marathon Lessons.

Tasks in the Lesson

Engage	8-10 minutes

During this part of the lesson, students will return to the ideas of the first lesson during which they worked with a number line model. Students will be introduced to the fraction bar model.

Post the set of problems they worked on during the previous lesson, and the examples of number line models. Remind students that they had worked to solve problems that involved finding groups of a certain size part.

Invite students to think about the essential question and let them know they will return to it at the end of the lesson: Essential Question: *What patterns do you notice in the products? How are the products related to the factors in the problems?*

Ask students to return to their observations from the end of lesson 1 when they discussed how the sizes of the products were related to the two factors.

Explore	18-20 minutes

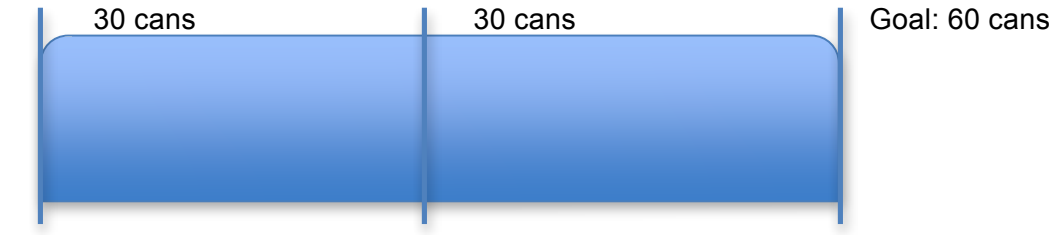
During this first part of the lesson, students will be working to make sense of the quantities in the problems and how they are related to the resulting amount. They will be using models and do not need to develop an algorithm or to notate their work with equations. During the discussion, there will be opportunities to link their models and ideas to the models from the previous lesson. These problems will allow students to think of multiplying a fraction by a whole number as finding a/b (part) of a whole number (the group).

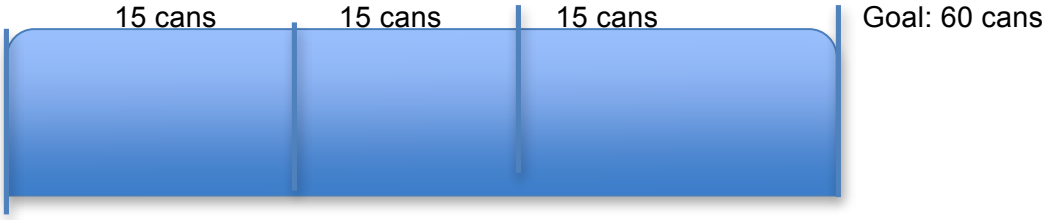
Introduce the story context: *A fifth grade class was collecting recyclables for a drive and then celebrated their success with an ice cream party. We are going to be solving problems about the drive and the party. We will be using fraction bars as models to solve the problems.*
Let's look a this first problem together: (Write the problem on a chart or board)

Stephen and Raul were looking at the how their class was doing on the recycling drive. They noticed that they were ½ way to their goal of 60 cans.

Introduce a bar model (note that this can be shown horizontally or vertically). Ask student where they think the boys saw the rectangle shaded for their class. Encourage students to explain how they know that the class has collected thirty cans Model the language ½ of 60 cans.

Continue the discussion by asking how many cans the class across the hall collected if they collected 1/3 of the 60 can goal?





Students work with partners or in small groups to solve the three recycling problems on *The Recycling Drive* activity sheet. As students work, ask them:

How does your fraction bar model show the total of cans? How did you decide how to divide your bar? Which landmark numbers were useful as you found parts of the bar?
Pose questions that ask:

What is half of____?
What is 1/10 of _____?
How does your drawing help someone see the part of the goal that the class has met? Is there something you could add or do to your model to make it clearer?

Explain	12-15 minutes

During this phase of the lesson, the teacher will select and sequence student strategies and engage the class in a discussion that compares the methods that students used to solve the problems. The discussion will begin to assist students in relating the actions of dividing the bars to the multiplication of a whole number by a fraction.

Bring students back together and discus the ways in which students used the fraction bars to solve the problems.

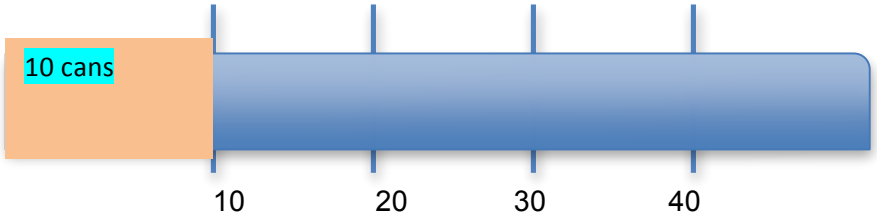
Begin with problem 1. Some students may have thought of first dividing the bar in half and notating 20 and 20 and then dividing the halves in half again. Some students may have divided the bar in fourths and thought of 40 divided by four.

As students share their ideas, listen for opportunities to talk about how the story contexts are asking for students to consider a part of a whole (the whole being the number of cans or bottles).

Move students to the notation of the fraction (part) x (of) whole number. For problem one, notate with the students ¼ x 40 and state: we were solving to find out what ¼ of 40 is.

Possible questions to pose: *How did the way you solve the problem show that you were finding ¼ of 40? Where do you see 4 parts? Where do you see how much ¼ of 40 is? How much does the whole bar represent?*

Possible Models:
Bar below shows 10 on first line, 20 on second 30 on third and 40 at end / shade in first part of bar to show that goal has reached 10 cans.



Discuss one additional problem taking time to discuss students' ideas, relate the ideas to the context and the models.

Return to the essential question.

Ask students what they are noticing about the products of these problems. Listen for statements that use language such as: *a part of , dividing the whole goal into parts, we took a fraction of.*

Allow students to build on each others' ideas as they begin to work toward this generalization: *the resulting product of a whole number and a fraction less than one will be greater than the fraction and less than the whole number factor*

Possible answers:
Since the classes didn't reach the whole goal the number of cans is always less than the goal.
Since we are looking at a part of the goal (the number) the number we get is less.

Elaborate 12-15 minutes

Students have been working with given story contexts in this lesson that ask them to consider finding a portion of a whole number.

During this phase of the lesson, students will be asked to write their own story contexts with a partner, and then exchange them with a new partnership. Each partnership will create a model for solving the problem, then the two partnerships will come back together to see how the models connect to the problems.

Suggested questions:

What is the same about your two stories/ What is different?
How do your models show the parts of your story contexts?

Evaluation of Students

Formative:

- Use the questions suggested above to assess student understanding as they interact with the problems and the models.
- Pose the essential question and ask students to respond in their journals. Assess which ideas of the lesson are in their responses.
- Student responses on the activity sheets will allow for assessing understanding of the ideas in the lesson.

Summative:

Students' work from the Elaborate section can be used as a summative assessment.

Plans for Individual Differences

Intervention: Some students may need an opportunity to work with whole numbers and the unit fraction ½ with several problems before moving to other fractions. While the numbers in these problems will be accessible for most students some students may need to work with smaller numbers.

Extension: Some students may benefit from additional opportunities for extending the work of the lesson. Ask students to write additional story contexts and prepare bar models that represent their problems.

Name_____ Date_____

The Recycling Drive

Draw fraction bars to find each class has contributed to the recycling contest.

1. Stephen's class has collected ¼ of the 40 bottles collected by the fifth grade classes. How many bottles have they collected?

2. Katie's class has collected 1/10 of 50 cans collected by the fifth grade classes. How many cans have they collected?

3. The fifth grade classes have filled 1/ 5 of the 25 recycling bins in the school. How many bins have they filled?

Lesson 2: Part 2: Collecting Recyclables

Overview and Background Information

Part 2: *Students will solve story problems collecting recyclables party and goal targets for a contest. They will incorporate the use of fraction bars and number lines. Students will begin to develop the following generalization: when you multiply two whole numbers, neither of which is zero, your product is always equal to or greater than each of the factors; when you multiply a fraction less than 1 by another fraction less than 1, the product is always less than either factor. The problems will be limited to a whole number multiplied by a fraction in the first part of the lesson and then to a fraction multiplied by a fraction in the second part.*

Mathematical Goals	By the end of the lesson students will: • Decompose fractions additively and related repeated addition to multiplication (4 x 1/3 = 1/3+1/3+1/3+1/3) • Relate partitioning and sharing contexts to fractions (division of numerator by the denominator) • Understand that the resulting product of a whole number and a fraction less than one will be greater than the fraction and less than the whole number factor • Interpret and create visual models for multiplying fractions (number lines and fraction bars) • Interpret and create story contexts for multiplying fractions Essential Question: *What patterns do you notice in the products? How are the products related to the factors in the problems?*
Common Core State Standards	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.3** Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.) **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
Emphasized Standards for Mathematical Practice	**2. Reason abstractly and quantitatively.** In this cluster of lessons students will have ample opportunities to engage in: "Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects." **7. Look for and make use of structure.** In this cluster of lessons students will have ample opportunities to engage in: "Mathematically proficient students look closely to discern a pattern or structure."

Prior Knowledge Needed	• Understand that fractions are numbers that represent quantities less than a whole • Understand the meaning of multiplication with whole numbers • Familiarity with number lines and bar models • Able to decompose fractions additively (3/4= 1/4+1/4+1/4) • 4NF4.Apply and extend previous understandings of multiplication to multiply a fraction by a whole number • Understanding of the distributive property with whole numbers.
Vocabulary	factor, product, fraction, fraction less than one, number line, fraction bars
Materials	Colored Pencils, Graph Paper, Fraction Bars, *About How Much* sheet, student sheet for Lesson 5.2.2 (Recycling Drive Part 2)
Resources	None needed

Tasks in the Lesson

Engage 5-7 minutes

During this part of the lesson, students will be introduced to a set of fractions that are not unit fractions. The launch will begin with an estimation activity that links the previous set of problems to this set.

Remind students of the previous lesson and ask students to share ideas that they recall from the lesson.

Invite students to think about the essential question and let them know they will return to it at the end of the lesson: Essential Question: *What patterns do you notice in the products? How are the products related to the factors in the problems?*
About How Much? (part 2) Begin by engaging students in an estimation routine. This launch will be used as an assessment and as an opportunity to model how estimating will be useful as students solve problems.

Show students the sequence of problems and ask them to estimate how the products will be changing.

Ask students: *How does finding ¼ of 20 relate to finding 2/4 of 20?*
How does 4/4 of 20 relate to finding ¼? To finding 2/4?

Use this opportunity to assess how students are decomposing the fractions to repeated additions of the unit fraction.

2/4 = ¼ + ¼ 4/4 = ¼ + 1/4 +1/4 +1/4

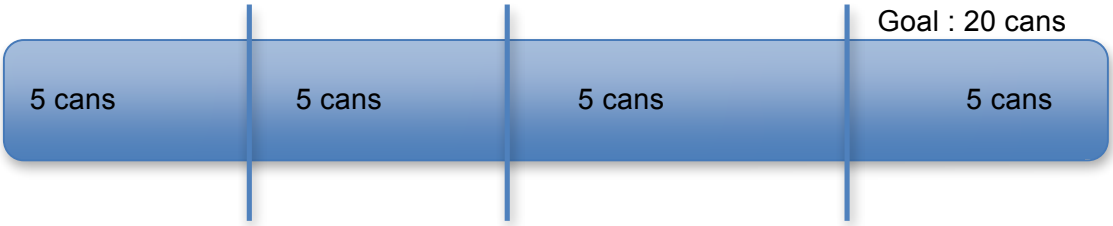
Explore	18-20 minutes

During this first part of the lesson, students will be working to make sense of the quantities in the problems and how they are related to the resulting amount. They will be using models and do not need to develop an algorithm or to notate their work with equations. During the discussion, there will be opportunities to link their models and ideas to the operation of multiplication. This will support the idea that b/d x a (b >1) is finding a part of a whole.

Introduce the following problem by writing it on a chart or on the board:

Mr. Chandler has asked each class to try to reach a goal of 20 cans by the end of the week. One fifth grade class collected ¾ of the goal.

Draw a bar model and ask students how many cans the whole bar represents (20 cans). Remind students how they showed ¼ of the bar in the earlier lessons and ask them how they would show that ¾ of the goal was met.



Possible questions to ask:

How many cans would we have if we collected ¼ of the 20 can goal be?
If we collected 2/4? If we collected ¾?
Can you tell me where there are three groups out of four groups in the diagram.
Why does each of the groups contain 5 cans?
Why would three groups of cans total 15 cans?

Students will work with partners or in small groups to solve the problems on the student sheet Recycling Problems Part 2. Note that all the problems require students to think about fractional parts of the total can goal which is 36.

As students work ask them: *How does your fraction bar model show the total of cans? How did you decide how to divide your bar? Which landmark numbers were useful as you found parts of the bar?*

Explain	15-18 minutes

During this phase of the lesson, students will share their strategies and make connections among the different methods that students used to solve the problems. The whole group discussion will focus on similarities between the problems in this set and the previous set of recycling problems.

Begin the whole group discussion with Problem

Ask for a student volunteer to draw the representation used to solve the problem. As students draw the bar and share their ideas, look for opportunities to discuss how they used their knowledge of unit fractions (1/4) and how they decomposed ¾ (¼+1/4+1/4).
Ask questions such as:

> *What does the whole bar represent? How many cans are represented in each of the four parts? How many cans in two parts? Three Parts?*

Write the equation: ¾ x 36 = 27 on the board/ chart. Ask students how their responses to the questions above connect to the equation you have written. *What does the 36 mean? What does it mean to have ¾ of 36 cans collected? Why is the result 27 smaller than 36?*

Ask students to turn and talk to a partner about problem number 2. *What would be an equation that would match your representation for problem number 2?* Ask for volunteers to share and connect their ideas to both the bar representation and the story context.

Return to the essential question: *What patterns do you notice in the products? How are the products related to the factors in the problems?*

Show the *About How Much* sheet from the beginning of the lesson. Ask students to assist you in recording equations for each of the problems:

> 1/4 x 20 = 5
> 2/4 x 20 =10
> 4/4 x 20 =20

Ask them what they notice about the size of the products and to explain the reason for the change in the products.

Possible Responses:
We always have a goal of 20 cans. The larger the fraction the more cans out of 20 we collected.
4/4 is a whole so we collected the whole number of cans which was 20.
We get a number less than 20 because we are only taking a part of 20.

During this discussion students will moving to the generalization:

- the resulting product of a whole number and a fraction less than one will be greater than the fraction and less than the whole number factor.

Elaborate	10-12 minutes

Ask students to work with partners or in small groups to write and represent additional recycling problems in which the can collecting goal was 60 cans.

After small groups have written their problems, ask them to share and compare their results.

Evaluation of Students

Formative:

a. Use the questions suggested above to assess student understanding as they interact with the problems and the models.

b. Pose the essential question and ask students to respond in their journals. Assess which ideas of the lesson are in their responses.

c. Student responses on the activity sheets will allow for assessing understanding of the ideas in the lesson.

Summative:

Students' work from the Explore section can be used as a summative assessment.

Plans for Individual Differences

Intervention: Some students may need an opportunity to work with whole numbers and the unit fraction ½ with several problems before moving to other fractions. While the numbers in these problems will be accessible for most students some students may need to work with smaller numbers.

Extension: Some students may benefit from additional opportunities for extending the work of the lesson. Ask students to write additional story contexts and prepare bar models that represent their problems.

Name_____ Date_____

The Recycling Drive Part Two

The contest is coming to an end and the fifth grade classrooms are discussing how close they came to meeting their can colleting goals for the last day. Use fraction bars to find out how close to the goals each class is. The goal was to collect 36 cans each day.

1. Ms. Wright's class has reached 3/ 4 of the can collecting goal.

2. Ms. Warren's class has collected 2/3 of the cans they needed to reach the goal.

3. At the end of the day, Mr. Garcia's class had collected 2/6 of the cans needed to make the goal.

Lesson 3: Servings at the Fifth Grade Ice Cream Party

Overview and Background Information

Students will solve story problems about ice cream cake servings at a fifth grade party and goal targets for a contest. They will incorporate the use of fraction bars and number lines. Students will begin to develop the following generalization: when you multiply two whole numbers, neither of which is zero, your product is always equal to or greater than each of the factors; when you multiply a fraction less than 1 by another fraction less than 1, the product is always less than either factor.

Mathematical Goals	By the end of the lesson students will: • Decompose fractions additively and related repeated addition to multiplication (4 x 1/3 = 1/3+1/3+1/3+1/3) • Relate partitioning and sharing contexts to fractions (division of numerator by the denominator) • Understand that the resulting product of two fractions less than one is less then either factor • Understand that the resulting product of a whole number and a fraction less than one will be greater than the fraction and less than the whole number factor • Interpret and create visual models for multiplying fractions (number lines and fraction bars) • Interpret and create story contexts for multiplying fractions Essential Question: *What patterns do you notice in the products? How are the products related to the factors in the problems?*
Common Core State Standards	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.3** Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.) **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
Emphasized Standards for Mathematical Practice	**2. Reason abstractly and quantitatively.** In this cluster of lessons students will have ample opportunities to engage in: "Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **7. Look for and make use of structure.** In this cluster of lessons students will have ample opportunities to engage in: "Mathematically proficient students look closely to discern a pattern or structure."

Prior Knowledge Needed	• Understand that fractions are numbers that represent quantities less than a whole • Understand the meaning of multiplication with whole numbers • Familiarity with number lines and bar models • Able to decompose fractions additively (3/4= 1/4+1/4+1/4) • **4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number • Understand that the resulting product of a whole number and a fraction less than one will be greater than the fraction and less than the whole number factor
Vocabulary	factor, product, fraction, fraction less than one, fraction bars, number lines
Materials	colored pencils, graph paper, fraction bars, *After the Party* activity sheet
Resources	None needed

Tasks in the Lesson

Engage 8-10 minutes

This lesson will have a variety of problems and students will be choosing to use number lines or bar models to solve the problems. During this part of the lesson the teacher will engage the students in thinking about the different models may make sense for certain story contexts. Students will be estimating the products that result from taking parts of parts. *The launch will begin with an estimation activity that links the previous set of problems to this set.*

Remind students of the previous lesson and ask students to share ideas that they recall from the lesson.

Invite students to think about the essential question and let them know they will return to it at the end of the lesson: Essential Question: *What patterns do you notice in the products? How are the products related to the factors in the problems?*

About How Much? (part 3) Begin by engaging students in an estimation routine. This launch will be used as an assessment and as an opportunity to model how estimating will be useful as students solve problems.

Show students the sequence of problems and ask them to estimate how the products will be changing.

Begin with the following:

Is 2 x 1 greater than or less than 1?
Is 1/2 x 1 greater than or less than 1?
Is 3/4 x 1 greater than or less than 1?

Ask students how the problems are related and how they are different.
Three of the problems have one as a factor so you have one group of a certain size (the problem with 2 as the size of the group will have the largest product)
Discuss how 2 x 1 is related to 2 x ½. *Two groups of 1 is more than 2 groups of ½ or if I have a goal of 2 cans and I meet the goal (multiply times one) I get more cans then if I have a goal of 2 cans and only meet half the goal.*

Discuss the remaining problems:

Is 1/2 x 1/2 greater than or less than 1?

Is 1/2 x 1/2 greater than or less than 1/2?

Use this opportunity to assess how students are thinking of ½ of ½.

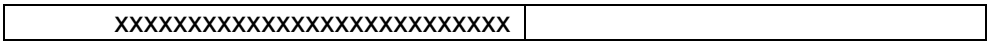
Ask: *What do you think it means to find ½ of ½?*

Explore 18-20 minutes

During this first part of the lesson, students will be working to make sense of the quantities in the problems and how they are related to the resulting amount. The students will be working to solve a problem that requires them to partition a number line or fraction bar and then to repartition a part of the bar. Only unit fractions will be used in these problems. Since students have been making sense of the notation in the previous lessons, it is helpful to begin writing the multiplication expressions next to the models as you work through the lesson.

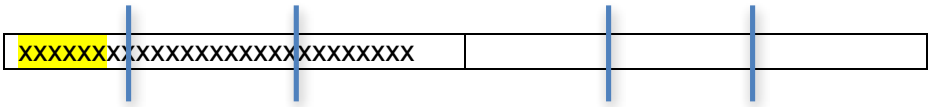
Write the following problem on the board or chart:
Ms. Johnson's class planned to collect ½ of their goal by Wednesday. On Tuesday morning they were 1/3 of the way to collecting ½ of their goal. How much had they collected?

Draw a fraction bar on the board or chart and shade in ½ the bar.



Ask the students what the shaded portion of the bar represents (*The shaded part represents ½ of the goal they want to reach by Wednesday.)* Ask students what the whole bar represents.

Pose the question: *How could we show that they have only collected 1/3 of the ½?* Students may say that they can divide or partition the shaded part into three parts. (Note: Students may not think of partitioning the un-shaded half as well/ discuss how it is important to keep the whole goal (bar) in mind as we think of parts of the parts).



What part of the whole goal have they met by Tuesday? Where in the bar model do you see 1 out of 3 parts in the shaded half?

How much of their goal have they met at that point? Where do you see 6 parts? Why does the yellow part show 1/6 and not 1/3?

It may be helpful to relate the bar to a whole number if students are not making sense of the six partitions. For example consider that that the goal is 30 cans and they plan to collect ½ by Wednesday (15 cans). If they collect 1/3 of the 15 cans by Tuesday, they have collected 5 cans which is 1/6 of their goal of 30 cans.

Students will now continue working on similar problems with a partner or in small groups. As students work, make note of which students are using number line models and which are using fraction bar diagrams. Do they consider the story context as they make choices about the models they are using? How are students considering the whole as they partition their diagrams? Ask students how their diagrams connect to their story contexts.

Explain	12-15 minutes

During this phase of the lesson, the teacher will select and sequence student strategies for solving the problems. The discussion will focus on what it means to find a part of a part and how the representations both show the whole, the part and the part of that part. Students will begin to notice that the result of multiplying a fraction by a fraction (both less than one) results in a fraction that is less than both fractions.

Begin the discussion with problem 1 on the *After the Party* handout.

The fifth grade collected ½ of the cans in the school recycling bins Mr. Chang's class collected ¼ of those cans. How much of the cans in the bins were collected by Mr. Chang's class?

As students share their solutions and representations ask:

How did you first partition or mark your bar/ number line?(two parts)

What did those parts show? (the whole divided into two parts) How did the number of parts match the denominator? How did you decide how much to shade? (one out of the two to show ½)

When you took ¼ of the half you broke each half into fourths. How many parts were in whole after that? (eight) What part of the fractions in the problem told you how to partition the bar/ number line? (the denominators)? What part of your representation shows how much was the of the cans were collected by the fifth grade? By Mr. Chang's class?

What equation could we write for this situation? (1/4x ½ =1/8)
Why is our product 1/8 smaller than the ¼ and the ½?

Choose another problem to continue a similar discussion.

Post the representation for problem number 5. Select two to three students to share their story contexts and discuss how they match or do not match the representation. Ask students what multiplication equation would represent the models/ story contexts(1/ 5 x 1 / 2 = 1 /10)

Return to the essential question: *What patterns do you notice in the products? How are the products related to the factors in the problems?*

On a chart record the multiplication expressions for each of the problems on which the students worked. Ask students what they notice about the products and the factors, and encourage students to make a general statement about what happens when you take a part of a part.

Elaborate	8-10 minutes

Students can pair up to share the story contexts they wrote for problem five and discuss how the representations match the stories they wrote.

Students can also create additional representations and switch with a partner who then writes a story context.

Evaluation of Students

Formative:

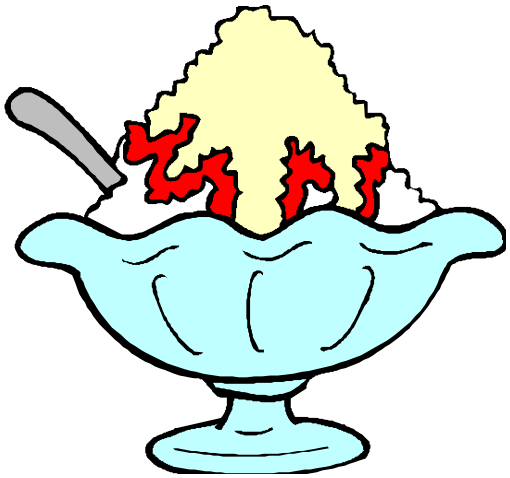
- Use the questions suggested above to assess student understanding as they interact with the problems and the models.
- Pose the essential question and ask students to respond in their journals. Assess which ideas of the lesson are in their responses.
- Student responses on the activity sheets will allow for assessing understanding of the ideas in the lesson.

Summative: Students' work from the Explore section can be used as a formative assessment.

Plans for Individual Differences

Intervention: Some students may need support in keeping track of the different partitioning moves they have made. Suggest using different colored pencils.

Extension: Students in need of an extension could solve an alternative task- instead of 1/3 of ½, they could look at 1/6 of 1/3 of ½.



About How Much?

Is 2 x 1 greater than or less than 1?

Is ½ x 1 greater than or less than 1?

Is ¾ x 1 greater than or less than 1?

Is ½ x 2 greater than or less than 1?

Is ½ x ½ greater than or less than 1?

Is ½ x ½ greater than or less than ½?

Name_____ Date_____

After the Party!

After the party there were some questions that the students and teachers wanted to answer. Solve each problem using number lines or fraction bars.

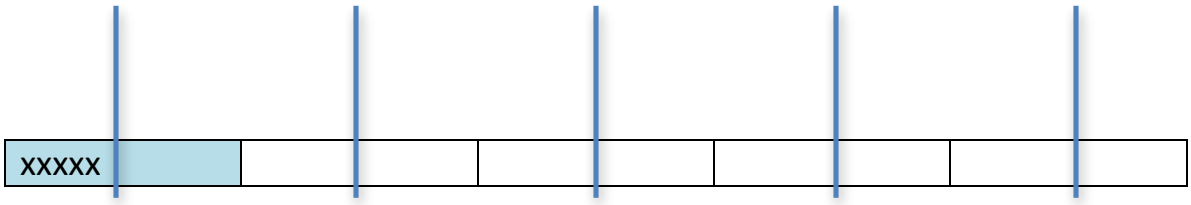
3. The fifth grade collected ½ of the cans in the school recycling bins Mr. Chang's class collected ¼ of those cans. How much of the cans in the bins were collected by Mr. Chang's class?

4. At the end of the ice cream party, there was 1/3 a container of whipped cream. The principal use ½ of that container for his sundae. How much whipped cream did he use?

4. The Community Center has large recycling bins for the neighborhood to use. Of all the bins, ¼ are for recycling cans.
   If Lance Elementary School filled 1/3 of the can bins, how much of all the bins did they fill?

4. The fifth grade teachers noticed that there was 1/5 of ½ gallon of strawberry ice cream left. How much strawberry ice cream was left?

5. Write a story context for this representation:



Lesson 4: Tiling the Walls of the Art Room

Overview and Background Information

Students will work within a story context about fifth grade students tiling the walls of the new art room at their school. They will make use of area models specifically arrays to make sense of the multiplication of unit fractions. This lesson continues the development of the generalization in the previous lesson with a different visual model (an area model).

Mathematical Goals	By the end of the lesson students will: • Interpret a story context involving multiplication of fractions • Represent a story context using an area model • Relate ideas about multiplication of whole numbers on arrays to multiplication of fractions less than one • Understand that unit squares represent a measurement of surface area Essential Question: *How do the lengths of a rectangle (array) correspond to the total area of the rectangle?* Return to EQ from previous lesson: *What patterns do you notice in the products when you multiply two fractions less than one?*
Common Core State Standards	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side/ Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.5** Interpret multiplication as scaling (resizing), by: **a.** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1. **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
Emphasized Standards for Mathematical Practices	**1. Reason abstractly and quantitatively.** In this cluster of lessons students will have ample opportunities to engage in: "Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects." **7. Look for and make use of structure.** In this cluster of lessons students will have ample opportunities to engage in: "Mathematically proficient students look closely to discern a pattern or structure."

Prior Knowledge Needed	Students will draw upon previous work with arrays with sides of whole number lengths. They will also make use of the ideas that a fractional amount originates from a whole and that when operating with two fractions it is understood that those two fractions originate from the same whole. Students will also use their knowledge and observations from the previous lesson as they continue investigation the size of the products.
Vocabulary	array, area, unit square, length, product, factor, fraction, fraction less than one
Materials	*Tiling the Art Room* handout, graph paper, tiles, white paper
Resources	Technology link: http://nlvm.usu.edu/en/nav/vlibrary.html See the manipulative for multiplying fractions with an array model

Tasks in the Lesson

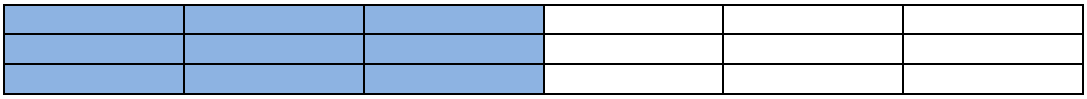
Engage 12-15 minutes

During this part of the lesson, students will have an opportunity to revisit an area model for multiplication of whole numbers. They will be engaged with a story context about a new art classroom whose walls will be tiled in a various ways. They will be introduced to a situation in which one of the dimensions of a tiling is a fractional amount.

Tell students that for the next two lessons they will be thinking about ways that a group of fifth grade students planned the tiling of a new art room at their school.

Show students an arrangement (array) of tiles. Ask the students to arrange the tiles in different ways so that there are always equal rows and columns (3x 6, 9x2 1x 18). Discuss how the rows and columns are part of the whole area (the 18 square units).

Show the following array and ask students what ½ of the area would be.



Discuss how the dimensions of this array are 3 by 6 and the whole array is 18 square units but that we can shade in half of the tiles/ squares or 9 square units.

Let students know that they will be working with arrays in the next part of the lesson but that the dimensions of the arrays will be fractional amounts.
Give each child a piece of white paper. Ask them to fold the paper horizontally into three equal parts (thirds). Tell them to choose a color and color one of the thirds.

Then ask students to fold the paper vertically in half and color one of the halves in a different color.

Ask: *What part of the paper has both colors? (1 out of the six parts or 1/6) Why did we end up with six parts? Why does only one of the parts have both colors? What are the dimensions of the two colored part? 1/3 by ½*

Let students know they will continue to explore these questions during the lesson. Pose the Essential Question: *How do the lengths of a rectangle (array) correspond to the total area of the rectangle?*

Explore 20-22 minutes

During this part of the lesson, students will consider how to partition a rectangle (the mural) into fractional parts and then find a part of those parts. They will represent parts of parts on with an area model. The idea of keeping track of all the parts in the whole is made explicit both in the diagrams and in the discussion of the values of the parts.

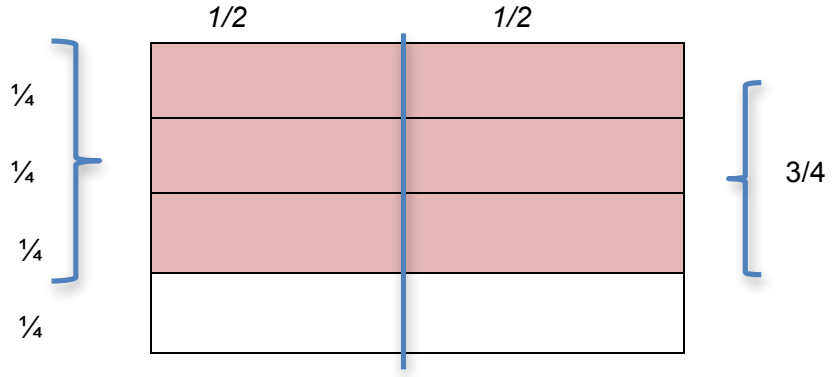
Pose this situation (write on board / chart)

The Lance Elementary School asked the fifth grade students to help the art teacher design some tile murals for the new art room. One of the murals was a 6 by 3 design like the one we just made with tiles. Another mural is going have ¾ of the design as red tiles and ½ of those will have flowers on them.

Draw a rectangle ask students how we could show that ¾ of a mural would have red tiles. *Partition a rectangle into four parts; the dimension of each part is ¼ by 1. Shade in 3 of the 4 parts (3/4) red*

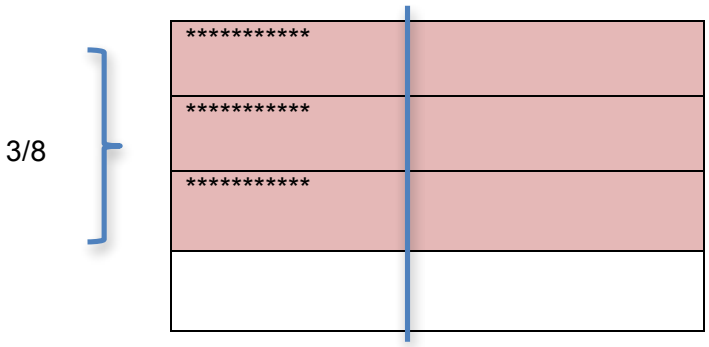
Ask students to share ideas for how they could represent that ½ of those red tiles would have flowers. *Divide each of the four sections in half; divide the whole mural in half.*

Ask : *How many parts has the whole mural been divided into? How do you know? What part of the mural are red tiles? (6/8 or ¾) Where do you see 6 out of 8 parts that are red?*



Show the above representation and ask students which part of the mural would have flowers. *½ of the red parts.*

Ask for a student volunteer to draw the flowers on the array.



Ask students:

*What part of the **whole mural** has flowers? (3/8 of the mural has flowers.) What does the 8 represent? What does the 3 represent? Which part of the mural were you finding? (We found ¾ of the mural and then we looked for ½ of that part) What multiplication equation does this array represent? (½ x ¾).*

Students will now work in small groups or with partners to solve additional problems about the murals in the art room. As students work pose questions such as the ones you posed in this part of the lesson.

Explain 18-20 minutes

During this phase of the lesson, the teacher will select and sequence student strategies for solving the problems. The discussion will focus on what it means to find a part of a part and how the ARRAY representations both show the whole, the part and the part of that part. Students will begin to notice that the result of multiplying a fraction by a fraction (both less than one) results in a fraction that is less than both fractions.

Begin the whole group discussion with Problem 2. Select students so that they may share their strategies for making the array representations and how the representation connects to the story context.

The students decided to create a tile arrangement with geometric shapes. 1/5 of the tiles will be triangles. ½ of the triangle tiles will be painted blue. What part of this mural will be blue triangles?

Continue the whole group discussion with a comparison of the representation for Problem 2 and Problem 3.

The students decided to create a tile arrangement with geometric shapes. 2/5 of the tiles will be triangles. ½ of the triangle tiles will be painted blue. What part of this mural will be blue triangles?

Pose questions:

How are the representations different? How are they similar? How do the two equations for the problems compare? (2/5 x ½= 2/10 and 1/5 x ½ = 1/10) Why is the product for the situation in Problem 2 twice as much as the product for Problem 1?

Return to the essential question: *How do the lengths of a rectangle (array) correspond to the total area of the rectangle?*

Student Sheet Art Room Part 2 will engage students in working with a mixed number multiplied by a fraction. Introduce problem 1 as written in the Explore portion of this lesson.

Elaborate 8-10 minutes

Students work to create story contexts and representations for given equations.

For students who are struggling give them a context and have them describe to you the situation, what numbers are important, and what the operations in the equation may be. After helping students write an equation for various contexts, see if they can then create a story context or picture for a given equation.

Evaluation of Students

Formative: While students are working, pose questions and observe them to check for their understanding.
Suggested things to observe or ask about:

- Can students correctly translate a story context into a picture and an equation?
- Can students clearly and accurately explain why they chose certain operations for fractions?
- Can students correctly translate an equation into a story problem or equation?

Summative: Students' work from all phases of this lesson could be used as a summative assessment?

Plans for Individual Differences

Intervention:
For students who need additional support, provide arrays that have one of the factors partitioned and labeled on the array and ask them to partition and label the second factor.
Extension:
The 3x6 dimensions are friendly since the area is 18. For students who are in need of an extension give them dimensions that will yield a total area that is an odd number, such as 3x7 or 3x9.

Name _____ Date _____

Tiling the Art Room

Use an array model to show how the fifth grade students completed the tile murals for their new art room. As you work, use what you know about arrays with whole numbers.

1. One of the murals in the art room will fit over the sink. This mural will have a pattern of light blue and black tiles. The black tiles will cover 2/3 of the design. The students will paint yellow suns on ¼ of those black tiles. What part of the whole mural will be black with yellow suns?

2. The students decided to create a tile arrangement with geometric shapes. 1/5 of the tiles will be triangles. ½ of the triangle tiles will be painted blue. What part of this mural will be blue triangles?

3. The art teacher asked the students to design a mural with their handprints. The students will cover 2/5 of the mural with handprints. ½ of those handprint tiles will be painted red. What part of this mural will be red handprints?

Name _____ Date _____

More Tiling of the Art Room

1. A large mural made up of handprint tiles will go on the left wall. This mural will measure 2 ½ feet by 4 feet. How large will the mural be?

2. The door of the art room measures 6 feet by 4 ½ feet. The art teacher is considering asking a group of fifth graders to paint the door with designs. How large an area will they be painting?

3. A small area above a window is available for a tiling design. The space measures 5 inches by ½ inch. How large is the area above the window?

Lesson 5: Who Won? The Lance School Relay Marathon

Overview and Background Information

During this lesson the students will make use of a linear model (number lines) and bar models. They will draw upon partitioning ideas as they repeatedly cut a whole into smaller, equal sized pieces.

Mathematical Goals	By the end of the lesson students will: • Use number lines and bar models to partition a whole into fractional pieces. • Communicate their processes and mathematical ideas
Common Core State Standards	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.5** Interpret multiplication as scaling (resizing), by: **a.** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **b.** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.
Emphasized Standards for Mathematical Practice	**1. Reason abstractly and quantitatively.** In this cluster of lessons students will have ample opportunities to engage in: "Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects." **7. Look for and make use of structure.** In this cluster of lessons students will have ample opportunities to engage in: "Mathematically proficient students look closely to discern a pattern or structure."
Prior Knowledge Needed	• Understand that fractions are numbers that represent quantities less than a whole • Understand the meaning of multiplication with whole numbers • Familiarity with number lines and bar models • Able to decompose fractions additively (3/4= 1/4+1/4+1/4) • 4NF4.Apply and extend previous understandings of multiplication to multiply a fraction by a whole number • Understand that the resulting product of a whole number and a fraction less than one will be greater than the fraction and less than the whole number factor
Vocabulary	factor, product, fraction, fraction less than one, fraction bars, number lines
Materials	adding machine tape, *Lance School Relay Races* handout, rulers
Resources	This site has a variety of visual fractions with which students can interact. http://www.visualfractions.com/index.htm

Tasks in the Lesson

Engage 12-15 minutes

During this part of the lesson, the teacher will engage the student with a story context about relay races. The students will create multiplicative situations and represent the "relay races" on strips of paper. They will later play a game with chips and number lines in which they consider the results of finding a part of a part.

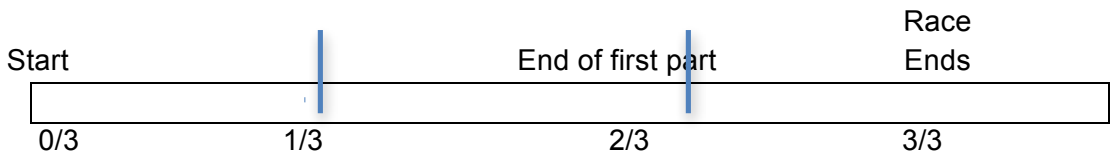
Let students know that a school is planning relay races for their fifth grade students. There will be different students running different parts of the race.

Write the following story on the board or on a chart:

> *Two students are running the first part of the race. They will run 2/3 of the track total. The first student begins at the starting point and will run ¼ of the first part of the race. Where is the second student standing?*

Show students a strip of adding machine tape and let them know the whole strip represents the racetrack (the whole race). Ask for a volunteer to show where the first part of the race will end. (*Partition the strip into three equal parts. Mark the end of the race at 2/3)*

Show the students that there are two chips to mark where the first student starts and where the second student is standing.



The first student begins at 0/3 or the start of the race. *How can we show that she has run the first part of the race?*

Explore 12-15 minutes

During this part of the lesson, students will work with their partners or small groups to construct relay race representations and write story contexts that match their representations.

Students will be working to create relay situations and then represent those situations on their strips. See the *Lance School Relay Race* activity sheet.

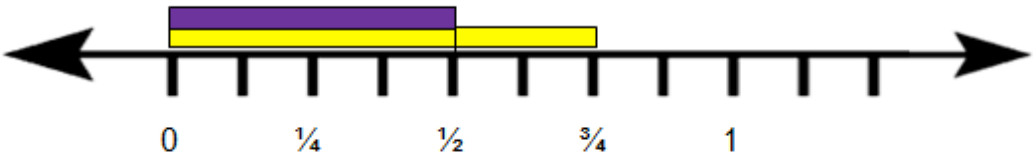
As students work observe how they:

- Partition the whole strip to show the first part of the race
- Decide how to partition the existing parts to show the portion of the relay that the first student ran
- Consider what part of the whole race the student has run

Explain 18-20 minutes

During this part of the lesson, students will be sharing their representations for the relay races and comparing the factors and size of the products to the work of other students.

Below is a possible diagram to show a relay of ¾ of the race where a student ran 2/3 of the race (6/12 or ½)



During the whole group share, focus the discussion on how the two factors(the length of the first part of the race and the length of the race the first student runs) are related to the where the second student stands on the track.

Use a table, such as the one below, to organize the students' work:

The first part of the race is ____ of the whole race.	The first runner runs ____of the first part.	The second runner is standing at ____ of the whole race.
½ of the whole race	2/3 of the ½ of the race	Is standing at 2/6 or 1/3 of the whole race
1/2	2/3	2/6
3/4	2/3	6/ 12

Pose these questions:

How does the numerator of the product relate to your diagram?
How does the denominator relate to your diagram?
What patterns do you see between the numerators and the denominators and their products? Why do you think those patterns appear?

Elaborate 10-15 minutes

Pose the following follow-up task:

When making a cake, the entire amount of cake batter is 5/6 of a gallon. Between 1/2 to 2/3 of the cake batter is flour. How much of the cake batter could be flour?

As students work on this, pose questions to help them.
Suggested questions:

- *What do you have to find?*
- *What representation can you use for this problem?*
- *Will we have one answer or a range of answers? Why?*

Evaluation of Student Work

Formative: While students work, observe them and pose questions to examine their mathematical understanding.

Summative: Students' work done throughout this lesson plan could be collected as a summative assessment.

Plans for Individual Differences

Intervention:
If students are struggling with the length model try using an area model for the opening task. In an area model students would show the problem ¼ of 2/3 by shading ¼ of a region that was partitioned into 4 parts using only vertical lines. The 2/3 will be shown by partitioning the same region into 3 parts by drawing horizontal lines. 2 of those 3 parts should be shaded. As a result of the picture there will be 12 smaller regions and 2 of the 12 will be shaded. 2/12 can be renamed as 1/6 which is the solution.

Extension:
If students need an extension, mixed numbers can be used to represent the beginning fraction for each of the tasks.

Name _____ Date _____

The Lance School Relay Races

Work with your partner or small group to create representations of the relay races. Your strip represents the entire race.

1. Choose what fraction of the race the first part of the race will be.
2. Choose how far the first student will run.
3. Decide where the second student will be waiting to continue the race.

1/2	1/3	1/4	1/5	1/6
2/3	2/4	3/4	2/5	3/5
4/5	2/6	3/6	4/6	5/6

Lesson 6: Sharing Prizes at the Spring School Carnival

Overview and Background Information

During this lesson students will first think about whole number division situations (sharing and grouping) and then division situations that involve fractions. They will then solve problems about sharing prizes that will develop models and strategies for dividing a whole number by a unit fraction and a unit fraction by a whole number.

Mathematical Goals	By the end of the lesson students will: • Understand that there are two problem types for division • Interpret division situations • Use representations to solve division of whole numbers by unit fractions • Use the inverse relationship between multiplication and division to solve division problems.
Common Core State Standards	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. **c.** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.
Emphasized Standards for Mathematical Practice	**2. Reason abstractly and quantitatively.** In this cluster of lessons students will have ample opportunities to engage in: "Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects." **7. Look for and make use of structure.** In this cluster of lessons students will have ample opportunities to engage in: "Mathematically proficient students look closely to discern a pattern or structure."
Prior Knowledge Needed	• Understanding of division of whole numbers • Interpretation of remainders • Able to represent sharing situations • Able to decompose fractions
Vocabulary	sharing, grouping, remainder, divisor, dividend, quotient Note: In the sharing situation, some known quantity (amount) is shared equally among a known number of entities (people, boxes, packages, etc.). What is not known in a sharing situation is the amount of the given quantity per share. The quotient in this situation represents the amount per share, the size of each share or the unit rate.

	In a grouping situation, the unknown is the number of groups of a given size that can be made from a given quantity (amount). The quotient in this situation tells how many groups of the specified size can be made from the given quantity.
Materials	*Prizes at the Spring Carnival* handout, fraction bars
Resources	This site has an interactive activity with fraction bars that allows students to visualize dividing fractions. http://nlvm.usu.edu/en/nav/category_g_3_t_1.html This site has a set of lesson plans that offer additional work with the goals of this lesson. http://www.utdanacenter.org/mathtoolkit/instruction/lessons/ 7divide.php

Tasks in the Lesson

Engage 12 to 15 minutes

During this part of the lesson, the students will compare and discuss two division situations. They will share diagrams for the two situations and then identify the two types of division problems: sharing or grouping (partitive or measurement/quotative). They will be presented with a story context about Prizes at a School Spring Carnival.

Begin the lesson by asking the students to identify situations that might involve sharing. Remind students that fractions are numbers that represent a division relationship

Ask students to work with a partner to represent the following two problems:

At the carnival there are 30 volunteers. We need volunteers at each of the five booths. How many volunteers will work at each booth?

The fifth graders face paint booth has 30 paintbrushes. We are putting the brushes in bags that hold 5 paintbrushes. How many bags do we make?

Questions to pose:

How can you illustrate the problem? What operation are you using?
Does the problem represent a sharing or grouping problem? Why do you think so?
What is the difference between a sharing and grouping problem?
What does the answer represent in a sharing problem? Grouping problem?
Can you give another example of a sharing problem? Grouping problem?

Let the students know that they will be working with both of these types of division problems over the next two days as they solve problems about the prizes at the school spring carnival.

Explore 18-20 minutes

During this part of the lesson, the students will work with partners to represent and solve a collection of problems. As students work, they will draw upon the various models they have developed for multiplying fractions as well as ones that have been useful for division of whole numbers.

Ask the students to consider the following problem:

During the carnival, Ms. Garcia notices that there 5 bags of balloons. She wants to give ½ a bag of balloons to some of the volunteers. How many volunteers can she give ½ a bag to?

Ask students to turn and talk to a partner and to be ready to explain if they think the number of volunteers that will get bags will be more or less than 5 and why.

Possible responses: *there are five full bags and since we are giving parts of the bags there will be more than five; there are two halves in one bag and four halves in two bags so it has to be more than five volunteers*

Ask a student volunteer to model the five bags with five fraction bars (wholes). Ask students how many halves they think are in five wholes and to work with a partner to draw or model with bars their solution.

Ask students to consider how they might use a number line to represent the five balloon bags and the ½ bags.
(a number line with five tick marks representing the five balloons and then jumps of size one half beginning at zero and landing on 5 for a total of 10 jumps)

Students begin working in small groups on a collection of problems. As students work, pose questions such as:

Where is the total number of_____ in your representation?
Which number represents the number of groups you have?
Which number represents the size of your groups?
How does the size of the quotient relate to the divisor?

Explain 12-15 minutes

During this part of the lesson, the teacher will select and sequence student sharing of strategies. The focus of the discussion will be on strategies for solving problems 3 and 4 followed by a comparison of the results for problems 3 and 4.

Bring the students back together and begin by discussing problem 3.

Discuss problem 4 and the strategies students used to solve the problem

Make sure to emphasize students' various representations, their strategies, and their rationale or reasons for the strategies that they chose.
Compare the two problems by asking questions:

How do the quotients differ? How can you explain how they differ? Could you write a problem that would give us a quotient that would differ in the same way for problem 1?

When we divide 5 by 1/4, there are four 1/4s in each whole bag. So we would multiply 5 x 4 (I have 5 wholes and each one has 4 fourths) to get 20. (Show this on a number line or with a bar model)

Pose the question: *If the problem is 5 divided by 2/4. How can we reason the quotient?*

Possible answers:
In this problem two 1/4s to make a prize bag. If we divide the 20 by 2 we get 10, which is the quotient. There are 10 2/4ths in 20.
Follow up by asking:

Will this reasoning work with all non-unit fraction divisors? Why?
When the divisor is a non-unit fraction what happens when you have a remainder?

Elaborate	8-10 minutes

Pose the following task:
When solving the task 15 divided by 1/3, your classmates have two different explanations.

Student 1: I know that 15 divided by 3 is 5. As a result I think the answer is 5.
Student 2: I think that if we have 15 cups of sugar in a jar and we were scooping the sugar with a scoop that was only 1/3 of a cup, then we would need 3 scoops to equal a cup. If we need 3 scoops per cup and we have 15 cups, then we need to scoop sugar 45 times.
Student 3: I know that 15 times 3 is 45. The answer is 45.

Is each student correct? Why or why not? What question would you ask each student about their answer?

Evaluation of Student Work

Formative: While students work, observe them and pose questions to examine their mathematical understanding.

Summative: Students' work done throughout this lesson plan could be collected as a summative assessment.

Plans for Individual Differences

Intervention: If students are struggling with the idea of dividing fractions, using manipulatives such as pattern blocks may help.

The hexagon pattern block could be a bag of balloons. The trapezoid (half of the hexagon) could be the amount that each volunteer gets. Students should see how many trapezoids they need in order to make 5 hexagons (bags of balloons).

Extension: If students need an extension, use a mixed number for the starting amount.

Name _____ Date _____

Prizes at the Spring Carnival

As you solve each problem, think about whether the problem is a sharing or grouping problem. How does your representation show the problem type?

1. During the carnival, the ring toss booth needed more prizes. The principal brought 4 boxes of stickers. Each student that came to the booth received 1/5 of the box. How many students will get prizes before they run out?

2. The popcorn booth serves tubs that weigh ¾ of a pound. I have a box that weighs 6 pounds. How many tubs can I make from the box?

3. The face-painting booth has ¼ pint of paint. There are five fifth graders who want their faces painted. How much paint should we use for each child?